# Optimal Property Taxation 

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#### Abstract

What is the optimal tax rate on residential housing? In this paper, I consider both the distributional effects and the long-lived transitional dynamics following a change in the property tax rate. To this end, I employ a life-cycle model calibrated to the U.S. economy, where asset holdings and labor productivity vary across households, and tax reforms lead to changes in house prices, wages, and the interest rate. The main result is that the optimal property tax is substantially higher than today. However, while a higher property tax is beneficial on average, almost all current homeowners are negatively affected. Time-varying policies show that there is hope of finding politically feasible policies that entail higher property taxes in the long run.


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## 1 Introduction

The remarkable increase in housing wealth has been a striking global trend over the last seventy years. House prices have by far outpaced income growth (see, e.g., Knoll et al. (2017)) and domestic capital accumulation has largely been driven by a rise in housing capital (Alvaredo et al. (2018)). The prominent role of housing has had wide-ranging implications for our view on, e.g., fiscal policy (Kaplan and Violante (2014)), monetary policy (Kaplan et al. (2018)), economic crises (Mian and Sufi (2011)), and inequality (Piketty (2014)). Hence, it is notable that we have yet to appreciate that an increasing share of the tax base comprises housing wealth and that this is likely to impact how governments should finance their expenditures. Actual tax policies also appear largely unaffected: property taxes as a share of GDP and total taxes have on average remained stable for the OECD countries since the mid-1960's (OECD (2021)).

This paper aims to broaden our understanding of residential property taxation. While property taxes are typically set at the local government level (OECD (2016)), I follow a long tradition in public finance that considers a consolidated government budget even if taxes are set at different government levels. In doing so, I disregard potentially important intergovernmental effects, but there are several reasons why this is a useful point of departure. First, the sheer size of housing wealth suggests that a comprehensive view of the property tax is warranted. Second, even if property taxes are set at a local level, they do have economy-wide implications as they affect, e.g., households' savings behavior.

To analyze how property taxes affect the economy at large, I use quantitative macroeconomic theory. I first look at a representative agent framework to better understand the long-run implications of changes to the property tax. However, the property tax also has distributional effects as, e.g., housing wealth varies substantially over the life cycle and within age cohorts. Thus, the main analysis is undertaken using a quantitative heterogeneous agent model. In the quantitative exercise, I use the U.S. as a laboratory for exploring optimal property tax rates. Hence, the results are easily comparable to those in the previous literature on optimal taxation, which mostly focuses on the U.S. A broad reform that would involve consolidation across different levels of government is not realistic in the U.S. at present, but it is still valuable to benchmark the current system against such a consolidated view. Moreover, a consolidated reform could be politically feasible in many other countries.

In more concrete terms, this paper sheds new light on the residential property tax by studying how it affects economic efficiency and households within and across generations. I first analyze the consequences of alternative property taxes in steady state, but the main analysis includes the long-lived dynamics following a policy change. To tie the results to
the literature on optimal taxation, and to structure the analysis, I search for property tax rates that maximize aggregate welfare either in terms of efficiency or utilitarian welfare. However, as households differ along many dimensions, it is not obvious how to aggregate individual welfare gains and losses. Thus, the goal is not to maximize aggregate welfare per se, as it involves taking a stance on the specific welfare weights. The aim is rather to carefully describe who wins and who loses from a reform. When investigating these reforms, I follow a common assumption in the literature by considering revenue-neutral policies. Since housing choices are intimately related to households' investment decisions, I assume that a linear capital income tax rate will adjust to keep revenues constant whenever a linear property tax changes.

I begin the analysis by showing that within a representative agent framework there are two strong theoretical arguments why the optimal steady-state property tax rate is substantially higher than today. First, efficient asset allocation is achieved by reducing the capital income tax to zero and financing the lost revenue with a higher property tax on rental and owner-occupied housing. By leaving capital income untaxed, the preferential tax treatment of owner-occupied housing under the current tax system is effectively removed as the after-tax returns to owner-occupied housing and other capital are equalized. ${ }^{1}$ Second, and along the lines of the Chamley-Judd result (after Chamley (1986) and Judd (1985)), it is better to tax housing consumption than capital income in steady state. In both cases, households benefit from higher wages due to an increased availability of business capital.

I move on to study optimal taxes through the lens of a quantitative life-cycle model with heterogeneous households. In the model, house prices, wages, and the interest rate respond endogenously to changes in the tax system. I first consider taxes that maximize aggregate welfare of newborn households in steady state, and I find that these align well with the theoretical predictions of the representative agent model. For example, the optimal policy in terms of aggregate efficiency involves an increase in the property tax from today's level of 1 percent to 6.1 percent and a decrease in the capital income tax rate from the current level of 36 percent to -0.8 percent. Although newborn households differ with respect to their labor productivity and asset holdings, 97 percent of households prefer the optimal policy to the current tax rates. Still, optimal policies do vary across households. Newborns with higher initial labor productivity also reap benefits from the current tax system as they can reduce their overall taxes by investing in owner-occupied housing. Thus, the optimal property tax decreases with households' productivity.

The steady-state analysis disregards several important transitional effects following a policy change. First, a sudden increase in property tax payments is likely to create both

[^1]winners and losers among current households. Second, while a higher property tax causes wages to rise, the increase is only gradual. It takes time for the capital stock to grow after a tax change is implemented. Third, both aggregate business capital and housing are relatively inelastic in the short run. Thus, it is ex ante unclear which of these assets is most desirable to tax for efficiency reasons.

To better understand how the transitional dynamics affect households' welfare, I consider the optimal one-time change in the property tax rate. ${ }^{2}$ The main finding is that the optimal property tax is considerably higher than today even after incorporating the long-lived transitional dynamics after a policy reform. The corresponding capital income tax is lower than its current rate but positive in the long run. For example, the optimal property tax for a utilitarian planner, who values equity as well as efficiency, is close to five times higher than today's level of one percent. This implies a capital income tax of about six percent in the long run compared to the current level of 36 percent. I show that these results are quantitatively robust to holding house prices fixed, halving the costs of transacting owner-occupied housing, or excluding the welfare effects of leaving bequests. Qualitatively, these results also hold for a planner who only cares about efficiency and when doubling the intertemporal elasticity of substitution, although the increase in the property tax is somewhat less pronounced in these two cases.

The optimal policies have large distributional effects and most of the current generations incur welfare losses from an increased property tax. Newborn generations along the entire transition benefit on average from higher property taxes. However, newborns that enter the economy closely after a policy change gain less as wages only increase gradually. Current homeowners, and especially retirees, are substantially hurt by increased property taxes and a sudden drop in house prices. ${ }^{3}$ As around two thirds of households own their home, a majority of current households do not want higher property taxes. In contrast, the welfare of almost all homeowners increases if the property tax is set to zero.

I show that a time-varying policy can increase the welfare of newborns in the long run and make most of the current households better off. Specifically, I consider a simple policy which first lowers the property tax rate to zero before it is increased substantially. Newborns in the long run benefit from such policies, mostly due to an increase in the wage level. The policy is of benefit to current homeowners by reducing their property tax payments and by increasing house prices relative to the once-and-for-all policy. These benefits are weighed against the costs of lower wages and higher capital income taxes in

[^2]the short run. I find that the benefits outweigh the costs for most homeowners if the property tax is kept low for an extended period. If the property tax is increased too rapidly, house prices do not increase sufficiently in the short run to compensate for the future cost of increased property taxes. Overall, these results are robust to different levels of the long-run property tax and allowing for a property tax that increases exponentially to its long-run level.

The time-varying policies I consider do, however, lead to welfare losses among many of the newborn generations along the transition to the new steady state. These welfare losses are large relative to the welfare gains among current generations. As a consequence, the aggregate discounted welfare of current and future generations tend to be considerably lower under time-varying policies as compared to the once-and-for-all policies.

Overall, this paper shows that even though standard welfare measures call for a higher property tax rate, the political economy of such changes are complicated. Moreover, it remains an interesting and open question whether the consolidated approach taken here is feasible in practice. I am hopeful that the findings in this paper motivate the search for more elaborate policies and that my framework will prove valuable in this pursuit.

This paper relates to the broader literature on optimal taxation. ${ }^{4}$ Within this literature, this paper is most closely connected to the relatively new strand of research that considers optimal capital income taxation with housing. Important contributions include Eerola and Määttänen (2013) who consider the interesting case of fully time-varying policies in a representative-agent model, and Borri and Reichlin (2021) and Nakajima (2020) who study optimal taxation with household heterogeneity in steady state. The main contribution of this paper is to study optimal property taxation with both an explicit account of the distributional effects and the transitional dynamics following a policy change. Furthermore, this paper relates to Guvenen et al. (2019) who find that taxing households' wealth rather than their capital income improves capital allocation by, e.g., increasing the savings rate of more productive households. Although a reallocation of capital also occurs in my model, this is achieved by incentivizing households to save in the "right" asset rather than incentivizing the "right" households to save. Finally, this paper complements a new line of research that studies time-varying policies using parametric approaches (see, e.g., Dyrda and Pedroni (2018) and Itskhoki and Moll (2019)).

The rest of the paper is organized as follows. I briefly discuss theories of optimal taxation in steady state in Section 2. In Section 3, I present a general equilibrium life-cycle model with heterogeneous households that captures salient features of the U.S. housing

[^3]market and tax system. The calibration of the model is then discussed in Section 4. In Section 5, I quantitatively study optimal steady-state taxes to better understand optimal taxes for newborns in the long run. The main results in this paper are discussed in Section 6 , which studies the optimal property tax rate when including the dynamics following a reform. A battery of robustness tests are discussed in Section 7. Finally, I provide some concluding remarks in Section 8.

## 2 Theoretical predictions in steady state

Before I turn to the quantitative model, let me first discuss two theoretical results of optimal steady-state taxation that push towards higher property taxes and a capital income tax of zero. First, it is better to tax housing consumption than business capital in steady state. Second, efficient asset allocation calls for an equalization of returns to housing capital and business capital.

### 2.1 Why tax housing consumption rather than business capital?

An important result in the literature on optimal taxation is the so-called Chamley-Judd result. According to this result, the capital income tax should be zero in the long run. ${ }^{5}$ A popular intuition for this finding is that a positive tax on capital income in the long run works as an ever-increasing tax on consumption, which cannot be optimal (see Judd (1999)). ${ }^{6}$

To study how the inclusion of housing impacts the original Chamley-Judd result, I adapt the discrete-time version of the Chamley model presented in Atkeson et al. (1999). ${ }^{7}$ Specifically, I separate consumption into non-housing consumption and consumption of housing services, and I allow for a tax on housing services and capital income. A detailed overview of the Chamley model with housing is presented in Appendix A.

[^4]The main takeaway from the analysis in Appendix A is that the Chamley-Judd intuition continues to hold in a model with housing. Thus, the capital income tax rate should be zero in steady state and government expenditures should be covered by taxing housing consumption. In particular, if households' utility from non-housing and housing consumption is given by a Cobb-Douglas utility function, which is what I am assuming in the quantitative analysis, this conclusion holds both in an infinite horizon model and in an overlapping-generations model.

### 2.2 Equalizing the return to housing and business capital

In addition to being another consumption good, housing is also a way for households to transfer wealth across time. As such, housing is a valuable alternative to financial savings for storing wealth. However, a common view is that a preferential tax treatment of housing distorts the relative return of these asset types in favor of housing. To understand this argument, it is useful to show exactly how the current tax system prefers savings in housing at the expense of business capital.

The preferential tax treatment of housing is easily shown by considering the net benefit of owning. The net benefit of owning is simply the difference between the cost of renting a house less the cost of owning a similar house. For simplicity, assume that rental units are provided by a rental company that incurs two costs: i) a financing cost $r$, where $r$ is the pre-tax return demanded by investors; and ii) a property tax $\tau^{h}$ per unit of housing. Then, based on a zero-profit condition, the rental price $p_{r}$ is

$$
p_{r}=r+\tau^{h} .
$$

For a homeowner, the flow cost $C_{o}$, i.e., the cost excluding transaction costs is

$$
C_{o}=\bar{r}+\tau^{h},
$$

where $\bar{r}=\left(1-\tau^{k}\right) r, \tau^{k}$ is a capital income tax, and $r$ is the return to investing in a financial asset. ${ }^{8}$ Thus, $\bar{r}$ captures the after-tax return that is foregone if a household chooses to buy a house. I abstract from the cost of mortgage financing to simplify the

[^5]exposition. Thus, the net benefit of owning is
\[

$$
\begin{equation*}
\mathcal{N}_{o}=p_{r}-C_{o}=\tau^{k} r . \tag{1}
\end{equation*}
$$

\]

Intuitively, $\tau^{k} r$ is the tax savings from not investing in the financial asset. It shows up because there is a tax on capital income, but no tax on the imputed rent of owner-occupied housing. As a result, the tax system incentivizes households to invest in owner-occupied housing rather than financial assets. A previous branch of the literature has emphasized that the preferential tax treatment of housing can be removed by taxing the imputed rent of owner-occupied housing (see, e.g., Gervais (2002) and Floetotto et al. (2016)). ${ }^{9}$ Clearly, an equally effective way to equalize the returns across asset types is to set the capital income tax to zero.

### 2.3 Why is a quantitative model required?

A main message from the theoretical exercise is that the optimal capital income tax in steady state is zero and that this implies an optimal steady-state property tax rate that is considerably higher than today. Yet, there are important features that are missing in the theoretical framework. A quantitative model can help fill these gaps.

First, results in the previous literature on optimal capital taxation have shown that model features such as borrowing constraints and earnings uncertainty can lead to other conclusions about the optimal capital income tax level (see, e.g., Aiyagari (1995) and İmrohoroğlu (1998)). Incorporating these features generally requires a quantitative framework.

Second, important features of housing have been omitted. Households make discrete choices on whether to rent or own. Households' ability to borrow is often limited by the amount of housing they own. Moreover, there are considerable transaction costs of buying and selling houses. The implications of including these model elements for optimal taxation are largely unknown, and these housing features are more easily included in a quantitative model.

Finally, the short-run welfare effects are more complicated than the steady-state consequences considered in the theoretical framework. One such complication is that most households alive at the time of a policy reform have already made decisions based on today's tax system. Thus, a tax reform is likely to create both winners and losers. Coupled with substantial heterogeneity among households with respect to age, labor income, house ownership, and financial assets, a tax reform may produce a wide dispersion of welfare

[^6]effects. These distributional consequences can be neatly captured in a quantitative heterogeneous agents model.

## 3 Quantitative model

To analyze optimal property taxes, I employ a general equilibrium life-cycle model with overlapping generations and incomplete markets. The model is in discrete time, where one model period corresponds to three years. There are five types of agents, namely, heterogeneous households, a representative production firm, a financial intermediary, a construction firm, and a government. Households enter the economy with unequal amounts of initial assets, face uncertain labor productivity during their working age, and are subject to an age-dependent probability of dying. They derive utility from non-housing consumption, housing services, and from leaving bequests. Housing services can either be obtained by renting from a financial intermediary or by owning a house. Housing purchases are considered to be long-term investments due to lumpy transaction costs of buying and selling houses. Households can thus save by investing in deposits or by building up housing equity. Borrowing is limited to homeowners and they have to adhere to a loan-to-value constraint. Proportional taxes decrease households' disposable income and are levied on labor income, capital income, and housing.

A representative firm produces goods using labor and capital as input, where labor is supplied inelastically by households and capital is borrowed from a financial intermediary. The intermediary also provides homeowners with mortgages and rents out housing services to tenants. Its operations are financed by households' deposit savings. The government operates a pay-as-you-go social security system, collects and distributes bequests, and taxes households, the financial intermediary, and the construction firm. The construction firm builds new housing based on the price of housing and a fixed amount of new land made available from the government.

In the benchmark model, the interest rate adjusts to clear the capital market, the house price adjusts to clear housing supply and demand, and the wage level and the price of rental housing are endogenous. The model is easily adapted to consider a case with fixed house prices. For ease of notation, I only write variables with subscripts for individuals $i$, age $j$, and time $t$ in cases where they are needed to avoid confusion.

### 3.1 Households

Demographics: The economy is populated by a measure one of households. Households can live at most 20 model periods, i.e., 60 years. They enter the economy at age $j=1$,
work until $j=J_{r}$ and cannot live past $j=J$. The probability of surviving between any two ages $j$ and $j+1$ is $\phi_{j} \in[0,1]$.

Endowments and labor earnings: Households have one unit of time available, which is supplied inelastically to the labor market. During working age, households face uncertain labor productivity, whereas households' time is unproductive during retirement. Specifically, the productivity of household $i$ at age $j$ is given by

$$
n_{i j}= \begin{cases}g_{j} \pi_{i j} & \forall j \leq J_{r} \\ 0 & \forall j>J_{r}\end{cases}
$$

where $g_{j}$ is a deterministic age-dependent component common across households, and $\pi_{i j}$ is a persistent productivity component. Specifically, the logarithm of the persistent component follows an $\mathrm{AR}(1)$ process

$$
\log \left(\pi_{i j}\right)= \begin{cases}\rho \log \left(\pi_{i, j-1}\right)+\nu_{i j} & \forall j \in\left\{2, \ldots, J_{r}\right\} \\ \nu_{i j}+\xi_{i} & \text { for } j=1,\end{cases}
$$

where $\rho \in[0,1]$ captures the persistence of productivity, $\nu_{i j}$ is an i.i.d. shock distributed $N\left(0, \sigma_{\nu}^{2}\right)$, and $\xi_{i}$ is an initial shock component with distribution $N\left(0, \sigma_{\xi}^{2}\right)$.

Pre-tax earnings are given by $y_{i j t}=w_{t} n_{i j}$ during working age, where $w_{t}$ is the wage level per labor-efficiency unit at time $t$. Retirement benefits are capped at $w_{t} \bar{s}$. Retirement benefits below the cap are given by $\tau^{r r} w_{t} n_{i J_{r}}$, where $\tau^{r r} \in[0,1]$ is the replacement rate and $n_{i J_{r}}$ is the productivity in the last working-age period. Formally, $y_{i j t}=w_{t} \min \left(\tau^{r r} n_{i J_{r}}, \overline{s s}\right)$ during retirement. A more detailed description of the productivity components and earnings is provided in Section 4.1.

Households are born with initial assets $a_{i 1 t}$ as in Kaplan and Violante (2014). During working age, households receive $a_{i j t}=\gamma_{t} w_{t} n_{i, j-1}$ in the form of bequests, where $\gamma_{t} \in[0,1]$ and $n_{i, j-1}$ is the labor productivity in the previous period. As labor is unproductive during retirement, retirees receive bequests as a fraction of their benefits, i.e., $a_{i j t}=\gamma_{t} y_{i j t}$ for $j>J_{r}$. In equilibrium, aggregate bequests received by households who are alive equal the amount left by households that die.

Preferences: Households derive instantaneous utility from a consumption good $c$ and housing services $s$. Formally

$$
U_{j}(c, s)= \begin{cases}e_{j} \frac{\left(c^{\alpha} s^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} & \text { if } \sigma>0, \sigma \neq 1  \tag{2}\\ e_{j}(\alpha \log (c)+(1-\alpha) \log (s)) & \text { if } \sigma=1\end{cases}
$$

where $e_{j}$ is an age-dependent equivalence scale that captures changes in household size
over the life cycle (see, e.g., Kaplan et al. (2020)), $\sigma$ is a parameter of relative risk aversion, and $\alpha$ is the expenditure share on consumption.

There is also a warm-glow bequest motive similar to that of De Nardi (2004), given by the bequest function

$$
U^{B}\left(q^{\prime}\right)= \begin{cases}v \frac{\left(q^{\prime}+w \bar{q}\right)^{1-\sigma}}{1-\sigma} & \text { if } \sigma>0, \sigma \neq 1 \\ v \log \left(q^{\prime}+w \bar{q}\right) & \text { if } \sigma=1\end{cases}
$$

where $v$ is the weight assigned to the utility from leaving bequests, $q^{\prime}$ is households' net worth, and $\bar{q}$ captures the extent to which wealthier households care more about leaving bequests relative to poorer households. ${ }^{10}$ For example, higher values of $\bar{q}$ mean that poorer households have less incentive to increase their net worth for the purpose of leaving bequests. As preferences are non-homothetic, there is a potential scaling issue: Whenever the wage level increases, poorer households save disproportionally more as $\bar{q}$ plays as a decreasingly smaller role. To remedy the scaling problem, I multiply $\bar{q}$ with the wage level $w .^{11}$ This way, preferences feature scale invariance in the aggregate, while I still allow for non-homothetic preferences in the cross section (see also the discussion in Mian et al. (2020)). The private discount factor is $\beta$ and the objective of households is to maximize the expected sum of discounted lifetime utility.

Deposits: Households can invest any non-negative amount in deposits $d^{\prime}$. The interest rate on deposits invested at time $t$ is $r_{t+1}$.

Houses: Housing services can either be obtained by owning a house or renting from the financial intermediary. Each unit of housing costs $p_{h, t}$ to buy and $p_{r, t}$ to rent. An owned house of size $h^{\prime}$ produces housing services through a linear technology $s=h^{\prime}$. These services have to be consumed by the owner of the house, which implies that households cannot be landlords.

Buying and selling owner-occupied housing is subject to transaction costs. The transaction cost of buying is $\varsigma^{b} p_{h, t} h^{\prime}$ with $\varsigma^{b} \in[0,1]$. Similarly, the cost of selling a house is $\varsigma^{s} p_{h, t} h$ with $\varsigma^{s} \in[0,1]$, where $h$ is the amount of owner-occupied housing a household enters the period with. Housing depreciates at the rate $\delta^{h} \in[0,1]$ in each period, and maintenance of $\delta^{h} p_{h, t} h$ must be paid by homeowners.

Housing is available in discrete sizes. ${ }^{12}$ The choice set of rental services is restricted to the ordered set of discrete sizes $S=\left\{\underline{\mathrm{s}}, s_{2}, s_{3}, \ldots, \bar{s}\right\}$. Owner-occupied housing is limited to a set $H$, where the smallest house size $\underline{\mathrm{h}}$ in $H$ is larger than the smallest available size

[^7]in $S .{ }^{13}$ Above and including that lower bound, both sets are identical.
Mortgages: Households can use mortgages $m^{\prime}$ to finance their homeownership. The interest rate on a mortgage taken up at time $t$ is $r_{t+1}^{m}=r_{t+1}+\kappa$, where $\kappa>0$. Mortgages are long-term and non-defaultable. Negative mortgage levels are not allowed, and a household cannot choose a positive level of mortgages in the last period $J$. The only other restriction is a loan-to-value (LTV) requirement which states that a household can only use a mortgage to finance up to an exogenous share $1-\theta$ of the house value
\[

$$
\begin{equation*}
m^{\prime} \leq(1-\theta) p_{h} h^{\prime} . \tag{3}
\end{equation*}
$$

\]

The LTV requirement is potentially binding for a household that takes up a mortgage when purchasing a new house or for a household that increases its current mortgage. A household that stays in its home and does not increase its mortgage is not subject to the LTV constraint.

Taxes: Households are subject to a range of linear taxes. Labor income is subject to both an income tax $\tau^{n}$ and a payroll tax $\tau^{s s}$ (only paid by working-age households, represented by the dummy variable $\left.\mathbb{I}^{w}\right)$. Both of these taxes are fixed throughout the analysis. For ease of notation, let $\bar{y} \equiv\left(1-\tau^{n}-\mathbb{I}^{w} \tau^{s s}\right) y$ denote after-tax labor income gross of deductions. Mortgage interest payments are deductible from labor income, which implies that the after-tax interest rate is $\bar{r}_{t}^{m} \equiv\left(1-\tau^{n}\right) r_{t}^{m}$. The return on deposits is subject to a capital income $\operatorname{tax} \tau_{t}^{k}$, which gives an after-tax return of $\bar{r}_{t} \equiv\left(1-\tau_{t}^{k}\right) r_{t}$. Lastly, the value of an owner-occupied house is subject to a property tax $\tau_{t}^{h}$ that is proportional to the house value. The capital income tax and the property tax are the only tax rates that will potentially vary across time.

Recursive formulation of the household problem: Households have one deterministic individual state: $j$ for age. They also have non-deterministic individual states, which I will denote $\mathbf{z} \equiv(n, x, h, m)$. Recall that $n$ is labor productivity, $h$ is the size of owner-occupied housing, and $m$ is the mortgage. The last state variable $x$ represents cash-on-hand and is defined as $x=\bar{y}+a$ for $j=1$ and

$$
x=\bar{y}+(1+\bar{r}) d-\left(1+\bar{r}^{m}\right) m+\left(\left(1-\varsigma^{s}\right)-\delta^{h}-\tau^{h}\right) p_{h} h+a
$$

for $j>1$. For computational reasons, and without any loss of generality, I define cash-onhand as including the net revenue of selling the house $\left(1-\varsigma^{s}\right) p_{h} h$. Households who do not sell their house between any two periods do not incur any transaction costs. Initial assets and inheritance are captured by the term $a$.

[^8]The household problem includes the discrete choice of whether to rent a home $(R)$, buy a house $(B)$, or stay in an existing house $(S)$. Then, for each household of age $j$ and living situation $k \in\{R, B, S\}$, the recursive problem can be formulated as follows:

$$
\begin{equation*}
V_{j, t}^{k}(\mathbf{z})=\max _{c, s, h^{\prime}, m^{\prime}, d^{\prime}} U_{j}(c, s)+\beta\left(\phi_{j} \mathbb{E}_{j, t}\left[V_{j+1, t+1}\left(\mathbf{z}^{\prime}\right)\right]+\left(1-\phi_{j}\right) U^{B}\left(q^{\prime}\right)\right) \tag{4}
\end{equation*}
$$

subject to

$$
\begin{aligned}
& c+d^{\prime}+\mathbb{I}^{R} p_{r, t} s+\mathbb{I}^{B}\left(1+\varsigma^{b}\right) p_{h, t} h^{\prime}+\mathbb{I}^{S}\left(1-\varsigma^{s}\right) p_{h, t} h \leq x+m^{\prime} \\
& q^{\prime}=\left(d^{\prime}+p_{h, t} h^{\prime}-m^{\prime}\right) /\left(\alpha+(1-\alpha) p_{h, t}\right) \\
& s=h^{\prime} \quad \text { if } h^{\prime}>0 \\
& h^{\prime}=0 \quad \text { if } k=R \\
& m^{\prime} \geq 0 \quad \text { if } h^{\prime}>0 \\
& m^{\prime}=0 \quad \text { if } h^{\prime}=0 \text { and/or } j=J
\end{aligned}
$$

and $c>0, s \in S, h^{\prime} \in H, d^{\prime} \geq 0$. The first constraint in the recursive problem is the budget constraint, where the left-hand side of the inequality is total expenditures and the right-hand side is the total funds available to spend. For all $k \in\{R, B, S\}$, a household chooses how much to consume $c$ and how much to save in deposits $d^{\prime}$. Additional costs occur depending on the specific living situation. In the renter case $\mathbb{I}^{R}=1$, the household needs to pay the cost of renting $p_{r, t}$. In the buyer case $\mathbb{I}^{B}=1$, the household needs to pay for the house purchase, which also includes a transaction cost. The total cost is thus $\left(1+\varsigma^{b}\right) p_{h, t} h^{\prime}$. As cash-on-hand $x$ is defined such that it includes the value of the house when sold, $\left(1-\varsigma^{s}\right) p_{h, t} h$ is added to the budget constraint as an expenditure in the stayer case, i.e., whenever $\mathbb{I}^{S}=1$. Households can cover their costs by spending their cash-on-hand $x$ or by borrowing $m^{\prime}>0$ whenever they buy or stay in an owner-occupied house. Stayers that increase their mortgage and buyers of new homes have to comply with the LTV constraint (3).

The second constraint in the recursive problem shows that the net worth $q^{\prime}$, which goes into the warm-glow utility function, is deflated by a price index $\alpha+(1-\alpha) p_{h, t}$. This captures the fact that any change in the house price affects the purchasing power of the agent that receives the bequests. The additional constraints are relatively standard. The solution to the household problem is given by

$$
V_{j, t}(\mathbf{z})=\max \left\{V_{j, t}^{R}(\mathbf{z}), V_{j, t}^{B}(\mathbf{z}), V_{j, t}^{S}(\mathbf{z})\right\},
$$

with the corresponding set of policy functions

$$
\left\{c_{j, t}(\mathbf{z}), s_{j, t}(\mathbf{z}), h_{j, t}^{\prime}(\mathbf{z}), m_{j, t}^{\prime}(\mathbf{z}), d_{j, t}^{\prime}(\mathbf{z})\right\} .
$$

### 3.2 Production

A representative firm uses capital $K_{t}$ and labor $N$ as inputs into a standard neoclassical production function to produce output goods $Y_{t}$. Formally,

$$
F\left(K_{t}, N\right)=Y_{t}=A K_{t}^{\alpha_{k}} N^{1-\alpha_{k}}
$$

where $A$ is aggregate productivity, $\alpha_{k}$ is the capital income share, and $N$ is the inelastic supply of labor. As usual the interest rate $r_{t}$ and wages $w_{t}$ are given by

$$
\begin{align*}
& r_{t}=A \alpha_{k}\left(\frac{N}{K_{t}}\right)^{1-\alpha_{k}}-\delta^{k}  \tag{5}\\
& w_{t}=A\left(1-\alpha_{k}\right)\left(\frac{K_{t}}{N}\right)^{\alpha_{k}} \tag{6}
\end{align*}
$$

where $\delta^{k}$ is the depreciation of capital.

### 3.3 Financial intermediary

There is a financial intermediary that operates as a bank and the sole provider of rental services. All deposits ( $D_{f, t}$ ) saved by households are invested in the intermediary at the interest rate $r_{t+1}$ and used to finance the intermediary's operations. The subscript $f$ indicates that the variable is specific to the financial intermediary. The intermediary provides mortgages to households, buys and rents out housing to households, and lends capital to the production firm. For simplicity, I assume that the intermediary only lives for two periods and earns zero profits.

Mortgages $\left(M_{f, t}\right)$ : Mortgage lending provides the intermediary with a net return of $r_{t+1}$. Although households pay an interest rate of $r_{t+1}^{m}=r_{t+1}+\kappa$, I assume that the mortgage spread $\kappa$ is a wasteful intermediation cost.

Capital $\left(K_{f, t}\right)$ : The net return on capital lending to the production firm is also given by $r_{t+1}$.

Rental Stock ( $H_{f, t}$ ): The gross return of rental operations is given by the rental income $p_{r, t}$ and accrues already in the first period. The operational costs comprise a depreciation cost $\delta^{h}$, an intermediation cost $\eta$, and a property tax $\tau_{t+1}^{h}$ that are all proportional to the value of the rental stock in the second period. Additionally, the intermediary incurs a financing cost $r_{t+1}$ as it uses deposits to finance the purchase of the
rental stock. After a tax reform is implemented, house prices may change. Let the capital losses per unit of the rental stock be $\Delta p_{h, t}=\left(p_{h, t}-p_{h, t+1}\right) / p_{h, t}$, i.e., if house prices fall capital losses increase. Expected capital losses and gains are reflected in the rental price, and will lead to higher and lower rental rates, respectively. The rental price that ensures zero profits is given by

$$
\begin{equation*}
p_{r, t}=\frac{1}{1+r_{t+1}}\left(\left(\delta^{h}+\eta+\tau_{t+1}^{h}\right) p_{h, t+1}+\left(r_{t+1}+\Delta p_{h, t}\right) p_{h, t}\right) . \tag{7}
\end{equation*}
$$

### 3.4 Housing supply and the construction firm

In the quantitative analysis, I study the importance of endogenous house prices for optimal taxation. I consider two cases. In the first case, I assume that the output good can be costlessly converted into housing capital. This implies that the relative price of housing is one and constant across policies. The model presented thus far can easily accommodate this assumption by setting $p_{h, t}$ to one. In the second case, which will be my benchmark model, I make the more realistic assumption that a construction firm produces housing capital and that its production depends positively on land availability and the relative price of housing. As a result, the output good is no longer assumed to be costlessly converted into housing capital and the relative price of housing is going to change across tax reforms.

I model the construction firm in a way that requires minimal changes to the current model framework and where the calibration of the model does not depend on which of the two housing supply formulations that is being considered. This makes it easier to pinpoint the effect of allowing for endogenous house prices.

I assume that at time $t$ a construction firm decides how much new housing capital to produce at time $t+1$, i.e., $I_{h, t+1}$. Specifically, investments in the housing stock at time $t+1$ takes the following reduced form

$$
\begin{equation*}
I_{h, t+1}=L p_{h, t+1}^{\epsilon} \tag{8}
\end{equation*}
$$

where $L$ is a fixed amount of new land made available every period, $p_{h, t+1}$ is the house price in period $t+1$, and $\epsilon$ is the elasticity of housing investment with respect to the house price. Following Favilukis et al. (2017), $L$ can be interpreted as a flow of government-issued permits. The extent to which newly available land is turned into actual housing units is then given by $p_{h, t+1}^{\epsilon}$. The higher the price and the higher the elasticity, the more housing is made available. The aggregate housing stock evolves according to

$$
\begin{equation*}
H_{t+1}=\left(1-\delta^{h}\right) H_{t}+I_{h, t+1} \tag{9}
\end{equation*}
$$

As the investment decision is made in the previous time period, this implies that the housing stock is perfectly inelastic in the period of an unexpected tax reform. ${ }^{14}$

The revenue from producing new housing capital $p_{h, t+1} I_{h, t+1}$ raises the question of how to distribute profits. I solve this issue by assuming that the government decides on a price per unit of land $\tau_{t+1}^{L}$ at time $t$ that the construction firm will have to pay at time $t+1$. Importantly, the price is set such that the expected profits of the construction firm is zero. ${ }^{15}$ As the firm's total costs are $\tau_{t+1}^{L} L$, the price per unit of land that ensures zero profits is $\tau_{t+1}^{L}=p_{h, t+1}^{1+\epsilon}$. In other words, all land is owned by the government and the government accrues all land rent. Thus, the property tax rate $\tau_{t+1}^{h}$ should be interpreted as a tax on the residential structure alone. In the first case, where housing capital is costlessly converted from the output good, $\tau_{t+1}^{L}$ is assumed to be zero for all $t$.

### 3.5 Government

The government runs a balanced pay-as-you-go (PAYG) retirement system, collects and redistributes bequests, and taxes the agents in a similar way as the U.S. tax system. ${ }^{16}$ The net tax revenues are spent on (wasteful) government expenditures $G$, which are assumed to be fixed throughout.

PAYG: The payroll tax $\tau^{s s}$ adjusts to make the PAYG system clear

$$
\begin{equation*}
\sum_{j=1}^{J} \Pi_{j} \mathbb{I}^{w} \int \tau^{s s} n_{j}\left(\mathbf{z}_{j}\right) d \Phi\left(\mathbf{z}_{j}\right)=\sum_{j=1}^{J} \Pi_{j}\left(1-\mathbb{I}^{w}\right) \int \min \left\{\tau^{r r} n_{J_{r}}\left(\mathbf{z}_{j}\right), \overline{s s}\right\} d \Phi\left(\mathbf{z}_{j}\right) \tag{10}
\end{equation*}
$$

where $\Pi_{j}$ is the age distribution of households with $\sum_{j=1}^{J} \Pi_{j}=1$ and $\Phi$ is the cross sectional distribution of the non-deterministic individual states at age $j$, i.e., $\mathbf{z}_{j}$. The left-hand side of equation (10) is the average payroll tax paid by all households. The right-hand side is equal to the average amount of pension benefits received by all households.

Bequests: The government collects bequests in the form of deposits and housing net of mortgages from households who die and redistributes the funds to newborns and surviving households. The net amount collected at time $t$ from a household that dies after

[^9]age $j$ is given by
\[

$$
\begin{aligned}
\mathfrak{q}_{j t}\left(\mathbf{z}_{j, t-1}\right) & =\left(1+r_{t}\right) d_{j, t-1}^{\prime}\left(\mathbf{z}_{j, t-1}\right)+\left(1-\varsigma^{s}-\delta^{h}\right) p_{h, t} h_{j, t-1}^{\prime}\left(\mathbf{z}_{j, t-1}\right) \\
& -\left(1+r_{t}^{m}\right) m_{j, t-1}^{\prime}\left(\mathbf{z}_{j, t-1}\right) .
\end{aligned}
$$
\]

The first term says that the government receives deposits plus any interest. The second term reflects the net amount received in terms of housing. Specifically, the government needs to pay the maintenance cost of the house before it sells the house and incurs the transaction cost of doing so. The last term shows that the government pays off any outstanding mortgages including interest. The total net amount collected is then

$$
\begin{equation*}
\mathfrak{q}_{t}=\sum_{j=1}^{J} \Pi_{j}\left(1-\phi_{j}\right) \int \mathfrak{q}_{j t}\left(\mathbf{z}_{j, t-1}\right) d \Phi\left(\mathbf{z}_{j, t-1}\right) \tag{11}
\end{equation*}
$$

Part of these bequests are distributed to newborns so that a newborn household has initial assets $a_{1 t}\left(\mathbf{z}_{1 t}\right)$ similar to those in the data, where the index 1 indicates period $j=1$. The remainder is given to households that are still alive. Recall that bequests received are $a_{j t}\left(\mathbf{z}_{j t}\right)=\gamma_{t} w_{t} n_{j-1}\left(\mathbf{z}_{j t}\right)$ for $j \in\left\{2, \ldots, J_{r}\right\}$ and $a_{j t}\left(\mathbf{z}_{j t}\right)=\gamma_{t} y_{j t}\left(\mathbf{z}_{j t}\right)$ for $j \in\left\{J_{r}+1, \ldots, J\right\}$. The parameter $\gamma_{t}$ adjusts such that

$$
\begin{equation*}
\mathfrak{q}_{t}=\sum_{j=1}^{J} \Pi_{j} \int a_{j t}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right), \tag{12}
\end{equation*}
$$

where $\mathfrak{q}_{t}$ is given by equation (11).
Taxes and expenditures: Total government expenditures $G$ are given by the government's tax revenues from households, the financial intermediary, and the construction firm as follows

$$
\begin{equation*}
G=\sum_{j=1}^{J} \Pi_{j} \int \Gamma_{j t}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right)+\tau_{t}^{h} p_{h, t} H_{f, t-1}+\tau_{t}^{L} L \tag{13}
\end{equation*}
$$

where taxes $\Gamma_{j t}\left(\mathbf{z}_{j t}\right)$ paid by households are

$$
\Gamma_{j t}\left(\mathbf{z}_{j t}\right)=\tau^{n}\left(y_{j t}\left(\mathbf{z}_{j t}\right)-r_{t}^{m} m_{j t}\left(\mathbf{z}_{j t}\right)\right)+\tau_{t}^{k} r_{t} d_{j t}\left(\mathbf{z}_{j t}\right)+\tau_{t}^{h} p_{h, t} h_{j t}\left(\mathbf{z}_{j t}\right) .
$$

Property taxes paid by the financial intermediary $\tau_{t}^{h} p_{h, t} H_{f, t-1}$ are levied on the rental stock bought by the financial intermediary in period $t-1$. Across policy reforms, the capital income tax $\tau_{t}^{k}$ adjusts to ensure that government revenues equal government expenditures.

### 3.6 Aggregate variables and market clearing

An aggregate resource constraint ensures that the agents in the economy do not spend more than what is available to them

$$
\begin{equation*}
C_{t}+p_{h, t} H_{t}+G+K_{t+1}+\Omega_{t} \leq Y_{t}+\tau_{t}^{L} L+\left(1-\delta^{k}\right) K_{t}+p_{h, t}\left(1-\delta^{h}\right) H_{t-1}, \tag{14}
\end{equation*}
$$

where $C_{t}$ is aggregate consumption, $H_{t-1}$ is the total housing stock at the beginning of time $t, G$ is government expenditures, $K_{t}$ is capital at the start of period $t, Y_{t}$ is total output, and $\Omega_{t}$ is the sum of the transaction costs related to buying and selling houses as well as the intermediation costs of mortgages and those related to the rental business. The income from producing housing capital enters the right-hand side of the resource constraint as $\tau_{t}^{L} L$, which is assumed to be zero when housing is costlessly converted from the output good.

Specifically, consumption is given by

$$
\begin{equation*}
C_{t}=\sum_{j=1}^{J} \Pi_{j} \int c_{j t}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) . \tag{15}
\end{equation*}
$$

$G$ is given by equation (13), whereas $H_{t}$ is

$$
\begin{equation*}
H_{t}=\sum_{j=1}^{J} \Pi_{j} \int s_{j t}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) . \tag{16}
\end{equation*}
$$

The (wasteful) transaction costs $\Omega_{t}$ associated with housing transactions, mortgage intermediation, and rental services are

$$
\begin{equation*}
\Omega_{t}=\Omega_{t}^{b}+\Omega_{t}^{s}+\Omega_{t}^{m}+\Omega_{t}^{\eta} \tag{17}
\end{equation*}
$$

The sum of the transaction costs related to housing purchases is given by $\Omega_{t}^{b}$, and is equal to $\sum_{j=1}^{J} \Pi_{j} \int \mathbb{I}^{B} \varsigma^{b} p_{h, t} h_{j t}^{\prime}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right)$, where again $\mathbb{I}^{B}$ is an indicator value equal to one for households that choose to buy a house. All houses that are bought end up being sold, either voluntarily or by the government upon death, which means that the transaction costs of selling are

$$
\begin{aligned}
\Omega_{t}^{s} & =\sum_{j=1}^{J} \Pi_{j} \int \mathbb{I}^{h^{\prime} \neq h \cap h>0} \varsigma^{s} p_{h, t} h_{j t}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) \\
& +\sum_{j=1}^{J} \Pi_{j}\left(1-\phi_{j}\right) \int \varsigma^{s} p_{h, t} h_{j, t-1}^{\prime}\left(\mathbf{z}_{j, t-1}\right) d \Phi\left(\mathbf{z}_{j, t-1}\right) .
\end{aligned}
$$

The first term is the transaction cost of selling for households that are alive, where $\mathbb{I}^{h^{\prime} \neq h \cap h>0}$ is an indicator value equal to one if a household decides to sell. The second term is the transaction cost for those who died between time period $t-1$ and $t$ and left owned housing behind. The cost of mortgage intermediation is $\Omega_{t}^{m}=\sum_{j=1}^{J} \Pi_{j} \int \kappa m_{j, t-1}^{\prime}\left(\mathbf{z}_{j, t-1}\right) d \Phi\left(\mathbf{z}_{j, t-1}\right)$. The total intermediation cost related to rental services is

$$
\Omega_{t}^{\eta}=\eta p_{h, t} H_{f, t-1},
$$

where $H_{f, t-1}$ is the amount of rental housing bought by the financial intermediary in period $t-1$.

Aggregate labor is fixed throughout and is given by

$$
\begin{equation*}
N=\sum_{j=1}^{J} \Pi_{j} \int n_{j}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) \tag{18}
\end{equation*}
$$

In equilibrium, capital demand $K_{t+1}$ from the production firm equals capital supplied by the financial intermediary at time $t$

$$
\begin{align*}
K_{t+1} & =K_{f, t}  \tag{19}\\
K_{f, t} & =D_{f, t}-\left(1-p_{r, t}\right) p_{h, t} H_{f, t}-M_{f, t} \tag{20}
\end{align*}
$$

where capital supplied $K_{f, t}$ departs slightly from models without housing as part of households' savings are used to fund the rental services provided to tenants $\left(1-p_{r, t}\right) p_{h, t} H_{f, t}$ and to cater to households' demand for mortgages $M_{f, t}$. The financial intermediary receives rental income immediately after it provides rental services to its tenants, and invests the income by lending to the production firm. Thus, only $\left(1-p_{r, t}\right) p_{h, t}$ per unit of the rental stock is effectively needed to cover rental-service operations. Aggregate deposits, the rental stock, and aggregate mortgages are given as follows

$$
\begin{align*}
& D_{f, t}=\sum_{j=1}^{J} \Pi_{j} \int d_{j t}^{\prime}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right)  \tag{21}\\
& H_{f, t}=H_{t}-\sum_{j=1}^{J} \Pi_{j} \int h_{j t}^{\prime}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) .  \tag{22}\\
& M_{f, t}=\sum_{j=1}^{J} \Pi_{j} \int m_{j t}^{\prime}\left(\mathbf{z}_{j t}\right) d \Phi\left(\mathbf{z}_{j t}\right) . \tag{23}
\end{align*}
$$

A formal equilibrium definition is relegated to Appendix C.

## 4 Calibration

### 4.1 Independently calibrated parameters

The model is calibrated to match salient features of the U.S. economy. Table 1 shows the full set of parameters that are based on estimates from the literature or computed based on data. Although a model period is three years, I show annualized values of the parameters to ease the interpretation when relevant.

Demographics: Households enter the economy at the age of $23-25(j=1)$. The last working period corresponds to the age group $62-64\left(J_{r}=14\right)$, and I assume that no household can live beyond the age group $80-82(J=20)$. The probability of dying between any two periods $j$ and $j+1$, i.e., $\phi_{j}$ is computed using the Life Tables for the U.S. social security area 1900-2100 (see Bell and Miller (2005)). Specifically, I use the observed and projected mortality rates for males born in 1950.

Endowments and labor earnings: The parameters related to labor productivity are based on the estimated earnings process in Karlman et al. (2021). Earnings and productivity levels map one-for-one as I set $w_{t}=1$ in the initial steady state. Specifically, I take the deterministic life-cycle profile of productivity $g_{j}$ to be the deterministic life-cycle earnings in their paper. The other parameters need some adjustments before they can be used. Indeed, the income process in Karlman et al. (2021) is assumed to consist of a household fixed effect, a transitory shock, and a permanent shock, whereas in this paper, I assume that productivity follows an $\mathrm{AR}(1)$ with an initial shock and a persistent shock. I set the persistence parameter $\rho$ such that the variance of $\log$ productivity is increasing roughly linearly up until retirement. I let $\sigma_{\nu}^{2}$ and $\sigma_{\xi}^{2}$ adjust such that the variance of log productivity for the age group 47-49 and the variance of log productivity for the age group $23-25$ are the same for the two processes. The age group $47-49$ was chosen since this is the period with the highest labor productivity.

Following Díaz and Luengo-Prado (2008), the replacement rate for retirees $\tau^{r r}$ is 50 percent. The maximum allowable benefit during retirement $\overline{s s}$ is calculated using data from the Social Security Administration (SSA) and corresponds to around 61 percent of average earnings for working-age households. The retirement benefits scale with $w_{t}$ as shown in Section 3.1, which means that the benefits received by retirees move with the wage level.

The initial asset holdings for households $a_{1}$ are calibrated as in Kaplan and Violante (2014). I divide households aged 23-25 in the Survey of Consumer Finances (SCF) into 21 groups based on their earnings. ${ }^{17}$ For each of these groups, I calculate the share with asset holdings above 1,000 in 2013 dollars and the median asset holdings conditional on having

[^10]| Parameter | Description | Value |
| :---: | :---: | :---: |
| Demographics |  |  |
| $J_{r}$ | Last working period | 14 (ages 62-64) |
| $J$ | Last possible period alive | 20 (ages 80-82) |
| $\phi_{j}$ | Survival probability | Bell and Miller (2005) |
| Endowments and labor earnings |  |  |
| $g_{j}$ | Deterministic labor productivity | Karlman et al. (2021) |
| $\rho$ | Persistence of prod. shock | 0.995 |
| $\sigma_{\nu}^{2}$ | Var of persistent prod. shock | 0.038 |
| $\sigma_{\xi}^{2}$ | Var of initial prod. shock | 0.119 |
| $\tau^{\text {r }}$ | Replacement rate retirees | 0.5 |
| $\bar{s}$ | Maximum benefit retirement | See text |
| $a_{1}$ | Initial assets | Kaplan and Violante (2014) |
| Preferences |  |  |
| $e_{j}$ | Equivalence scale | See text |
| $\sigma$ | Coefficient of relative risk aversion | 2 |
| Houses |  |  |
| $\varsigma^{b}$ | Transaction cost buying house | 0.025 |
| $\varsigma^{s}$ | Transaction cost selling house | 0.07 |
| $\delta^{h}$ | Depreciation, housing | 0.023 |
| $\epsilon$ | Elasticity of housing investments | 1.5 |
| Mortgages |  |  |
| $\theta$ | Down-payment requirement | 0.20 |
| $\kappa$ | Yearly spread, mortgages | 0.01 |
| Taxes |  |  |
| $\tau^{k}$ | Capital income tax | 0.36 |
| $\tau^{h}$ | Property tax | 0.01 |
| Production |  |  |
| $r$ | Interest rate | 0.066 |
| $\delta^{k}$ | Depreciation, capital | 0.067 |
| $\alpha_{k}$ | Capital income share | 0.265 |
| $w$ | Wage | 1 |
| A | Aggregate productivity | 1.4 |

Table 1: Independently calibrated parameters, based on data and other studies
Note: The values are annual for the relevant parameters. When simulating the model, I adjust these values to their three-year (one model period) counterparts.
assets above this limit. The median asset value for each group is scaled by the median earnings among working-age households (23-64) in the SCF data. For model purposes, I rescale these asset values with the median earnings of working-age households in my model. Since the initial assets are scaled by earnings, they will move with changes in the wage level.

Preferences: The equivalence scale $e_{j}$ is equal to the square root of the predicted
values from a regression of family size on a third-order polynomial of age. Predicted values were obtained using data from the Panel Study of Income Dynamics (PSID) for the years 1970-1992. In the benchmark model, I set the coefficient of relative risk aversion $\sigma$ to 2, a standard value in the literature.

Houses: The transaction costs of buying and selling a house are taken from Gruber and Martin (2003), who estimate these costs to around 2.5 and 7 percent of the house value, respectively. Based on data from the Bureau of Economic Analysis (BEA), covering the years 1989-2013, I set the depreciation rate of owned housing to 2.3 percent. Similar to Kaplan et al. (2020), I set the elasticity of housing investment with respect to the house price $\epsilon$ to 1.5.

Mortgages: The minimum down-payment requirement when purchasing a house or increasing an existing mortgage is set to 0.2 , which is a standard value in the literature. I choose a yearly spread for mortgages $\kappa$ of 0.01 . This is approximately the spread between the contract rate on 30-year fixed-rate conventional home mortgage commitments and market yields on the 30-year constant maturity nominal Treasury securities over the period 1997 to 2015.

Taxes: Following Trabandt and Uhlig (2011), I let the initial capital income tax rate $\tau^{k}$ be 0.36 . This is broadly in line with what papers in the optimal capital taxation literature have been using. Acikgöz et al. (2018), Davis and Heathcote (2005), Domeij and Heathcote (2004), and İmrohoroğlu (1998) all used a capital income tax rate in the range of $0.36-0.4$. The key tax rate in this paper is the property $\operatorname{tax} \tau^{h}$, which is 0.01 in the initial economy. This is based on data from the 2013 American Housing Survey (AHS), which show that the median amount of real estate taxes per $\$ 1,000$ of housing value is approximately 10 dollars. ${ }^{18}$

Production: The interest rate $r$ is equal to the rental rate of total capital $R^{T}$ less the depreciation of total capital $\delta^{T}$. Assuming a Cobb-Douglas production function for the total economy, the rental rate is equal to $\left(Y^{T} / K^{T}\right) \alpha_{k}^{T}$, where $Y^{T}$ is the gross domestic product (GDP) less investments in defense-related capital, $K^{T}$ includes all non-defense capital, i.e., both residential and nonresidential capital, and $\alpha_{k}^{T}$ is the capital income share for total capital $K^{T}$ which I assume to be $1 / 3$. Using data from the BEA for the years 1997 - 2013, I find that the rental rate of total capital $R^{T}$ was 0.117 on average. The depreciation rate $\delta^{T}$ is 0.051 and is computed as the depreciation of total capital divided by total capital. Overall, the values for the rental and depreciation rates imply an interest rate of 0.066 .

To compute $\delta^{k}$, the depreciation rate for production capital in my model, I divide the depreciation of nonresidential capital by the stock of nonresidential capital. This gives a

[^11]yearly depreciation rate of 0.067 .
The capital income share $\alpha_{k}$ for the production capital in my model is computed as $\left(R^{N} K^{N}\right) / Y^{N}$, where $R^{N}=r+\delta^{k}$ is the rental income of nonresidential capital, $K^{N}$ is nonresidential capital, and $Y^{N}$ is GDP $\left(Y^{T}\right)$ less consumption of housing services. I assume that the return net of depreciation $r$ is the same for all capital types. Then, the capital income share is easily computed and it is equal to 0.265 . Thus, the capital income share for nonresidential capital is slightly lower than that for total capital.

Aggregate productivity $A$ can be computed using the equations for the interest rate (5) and the wage level (6). First, solve (5) for $K_{t} / N$ and substitute into (6). Second, impose $w_{t}=1$ and solve for $A$ to get

$$
A=\left(\frac{1}{1-\alpha_{k}}\right)^{1-\alpha_{k}}\left(\frac{r+\delta^{k}}{\alpha_{k}}\right)^{\alpha_{k}}
$$

Since $\alpha_{k}, r$, and $\delta_{k}$ are known, $A$ is also known and equal to 1.4.

### 4.2 Internally calibrated parameters

Table 2 shows parameters internally calibrated by simulation, along with a comparison between data and model moments. ${ }^{19}$ Unless otherwise stated, I use data from the SCF.

| Parameter | Description | Value | Target moment | Data | Model |
| :---: | :--- | :---: | :--- | :---: | :---: |
| Preferences |  |  |  |  |  |
| $\alpha$ | Consumption weight in utility | 0.82 | Median house value-to-earnings | 2.32 | 2.32 |
| $v$ | Utility shifter of bequest | 1.3 | Share of net worth held by $j=J$ | 0.03 | 0.02 |
| $\bar{q}$ | Luxury parameter of bequest | 2 | Homeownership rate, age $74-82$ | 0.80 | 0.79 |
| Houses |  |  |  |  |  |
| $\eta$ | Intermediation cost, rentals | 0.031 | Homeownership rate, age $<35$ | 0.43 | 0.34 |
| $\underline{\mathrm{~h}}$ | Minimum owned house size | 39 | Homeownership rate | 0.68 | 0.68 |
| Taxes |  |  |  |  |  |
| $\tau^{n}$ | Labor income tax | 0.12 | Gov. consumption to GDP $(G / Y)$ | 0.17 | 0.17 |
| Equilibrium | objects |  |  |  |  |
| $\beta$ | Discount factor | 0.97 | Asset market clearing | See text |  |
| $\gamma$ | Bequest rate | 0.09 | Bequest clearing | See text |  |

Table 2: Internally calibrated parameters
Note: Parameters calibrated either by simulation or as the result of equilibrium conditions. The third column shows the resulting parameter values from this estimation procedure. The values are annual when applicable. When simulating the model, I adjust these parameter values to their three-year (one model period) counterparts. The fifth column presents the values of data moments that are targeted. The last column shows the model moments that are achieved by using the parameter values in column three.

Preferences: The parameter $\alpha$ determines the weight on consumption and housing

[^12]services in the utility function. I use this parameter to calibrate the median house value relative to earnings, conditional on owning a house. The strength of the bequest motive $v$ affects how much net wealth households want to leave behind if they die. Thus, I calibrate it to target the share of net worth held by households in the last period. The other bequest parameter $\bar{q}$ determines the extent to which bequests are luxury goods, and it will affect the fraction of households who would want to remain homeowners as they age. For this reason, I calibrate $\bar{q}$ to target the homeownership rate among those who are between 74 and 82 years old.

Houses: I set the intermediation cost of rental housing $\eta$ to target the homeownership rate for those aged below 35, as it affects how early in life households become homeowners. For example, a higher value of $\eta$ increases the cost of rental units relative to owner-occupied housing and will, all else equal, increase the homeownership rate for the young. The minimum owner-occupied house size $\underline{h}$, which corresponds to roughly twice the average annual earnings for working-age households in the model, is calibrated to match the overall homeownership rate.

Taxes: I let the tax rate on labor income $\tau^{n}$ adjust such that $G / Y$ is 0.17 , which was the average value of government consumption-to-GDP over the years 1989-2013 based on data from the BEA. For GDP, I exclude investments in national defense and consumption of housing services.

Equilibrium objects: The discount factor $\beta$ is an equilibrium object in the initial steady state. Specifically, $\beta$ affects how much households save and adjusts to ensure that capital supply $K_{f}$ equals capital demand $K$, where the latter is fixed in the initial steady state as $r$ is taken from data. In all other steady states, the discount factor is held constant and the interest rate varies. The bequest rate $\gamma$ is also an equilibrium object and it is the solution to the bequest scheme given by equation (12). The value of $\gamma$ will vary with the different policy experiments.

## 5 Optimal property taxation in steady state

This section studies optimal taxation in steady state, i.e., without considering the impact on current generations and the transition to a new steady state. This is a natural starting point for several reasons. First, the theoretical arguments in Section 2 for taxing housing more heavily than today are based on steady-state considerations. I show that these arguments also play an important role in my quantitative analysis. Second, a social planner is likely to assign some weight to future generations. In the limit, as the social discount factor approaches 1 , the welfare consequences for newborns in the long run will dominate. For these newborn generations, the optimal policy is given by the optimal
steady-state reform. Finally, to understand why short-run welfare effects differ from those in steady state, it is clearly helpful to have an idea of the factors that determine the welfare effects in steady state.

### 5.1 Welfare measure and planner problem

In order to compare steady-state policies, I need an interpretable measure of welfare. There are two potential difficulties when choosing a suitable welfare measure. First, one needs to decide how to measure individual welfare. Second, individual welfare must be aggregated to a measure of societal welfare. This aggregation is non-trivial as households differ in many respects.

I measure individual welfare as the constant consumption stream that is equivalent to a household's ex-post value function. To be more concrete, let the value function $V_{i}\left(\tau^{h}\right)$ for newborn $i$ under a policy with the property tax rate $\tau^{h}$ be

$$
V_{i}\left(\tau^{h}\right) \equiv \sum_{j=1}^{J}\left(\beta^{j-1} \prod_{k=1}^{j-1} \phi_{k}\right)\left[U_{j}\left(c_{i j}\left(\tau^{h}\right), s_{i j}\left(\tau^{h}\right)\right)+\beta\left(1-\phi_{j}\right) U^{B}\left(q_{i j}^{\prime}\left(\tau^{h}\right)\right)\right],
$$

where $c_{i j}\left(\tau^{h}\right), s_{i j}\left(\tau^{h}\right)$, and $q_{i j}^{\prime}\left(\tau^{h}\right)$ are the realized values of consumption, housing services, and net worth of household $i$ at age $j$ under policy $\tau^{h}$. Moreover, $\beta^{j-1} \prod_{k=1}^{j-1} \phi_{k}$ is the effective discount factor for streams of utility at age $j$ from the perspective of a newborn. Next, let $\omega_{i}\left(\tau^{h}\right)$ capture individual welfare in consumption equivalents under policy $\tau^{h}$. Specifically, $\omega_{i}\left(\tau^{h}\right)$ is the constant consumption stream for individual $i$ such that

$$
\begin{equation*}
V_{i}\left(\tau^{h}\right)=\sum_{j=1}^{J} \tilde{\beta}^{j} \frac{\omega_{i}\left(\tau^{h}\right)^{1-\sigma}}{1-\sigma}, \tag{24}
\end{equation*}
$$

where households' effective discount factor is $\tilde{\beta}^{j} \equiv e_{j} \beta^{j-1} \prod_{k=1}^{j-1} \phi_{k}$.
I aggregate individual welfare $\omega_{i}\left(\tau^{h}\right)$ as follows

$$
\begin{equation*}
\left(\int_{0}^{1} \omega_{i}\left(\tau^{h}\right)^{1-\hat{\sigma}} \mathrm{d} i\right)^{\frac{1}{1-\hat{\sigma}}} \tag{25}
\end{equation*}
$$

where the desire to redistribute is given by the planner's inequality aversion $\hat{\sigma}$. The higher is $\hat{\sigma}$, the higher is the desire to redistribute. I study two cases. As a first aggregate measure, I consider aggregate economic efficiency by assuming $\hat{\sigma}=0$. Thus, aggregate efficiency is the average of individual consumption equivalents. As in Benabou (2002), redistribution that, e.g., reduces consumption fluctuations increases efficiency. However, redistribution for pure equity considerations is not deemed efficient. As a second aggregate measure, I consider utilitarian welfare by assuming that $\hat{\sigma}$ equals households' risk aversion $\sigma$. In

Appendix B.1, I show why utilitarian welfare takes this specific form. Overall, the welfare measures allow for a clear mapping between individual welfare and aggregate welfare. Moreover, the welfare measures allow for a convenient distinction between efficiency and equity concerns.

The planner problem is to choose $\tau^{h}$ to maximize (25) subject to a value of $\hat{\sigma}$ and a series of other restrictions. First, the government constraint (13) needs to hold. As government spending $G$ is assumed to be fixed, I let the capital income tax rate $\tau^{k}$ adjust to ensure that the government's net revenues equal $G$. I do not allow the government to borrow or lend. Second, the amount of bequests left should equal the amount received by households. The bequest parameter $\gamma$ adjusts such that bequests balance under all policies. Third, any equilibrium must be a competitive equilibrium, where the interest rate adjusts to ensure that capital demand by the production firm equals capital supply. Finally, housing demand must equal housing supply. For the benchmark results, I assume that the house price adjusts to clear the housing market. I also consider a case with fixed house prices, or equivalently perfectly elastic housing supply, in which the housing market clears automatically.

### 5.2 Quantitative results in steady state

In line with the theoretical results in Section 2, I find that the optimal steady-state property tax is much higher than the current level of one percent (see Figure 1a). Figure 1 lb shows that this implies an optimal capital income tax close to zero, which is considerably lower than today's level of 36 percent. For example, a planner who maximizes efficiency optimally chooses a property tax of about six percent and a capital income tax that is only slightly negative ( -0.8 percent). The average welfare gain is large. Moving to the optimal property tax level entails a 3.4 percent increase in the average of consumption equivalents relative to the current tax system. Similar results hold for a planner who maximizes utilitarian welfare.

The optimal tax system gives rise to a considerable reallocation of capital which increases GDP. Table 3 compares aggregate variables and prices between the initial economy and the optimal utilitarian steady state. Many households delay their housing purchases and allocate more of their funds to deposits. On average, households increase their deposit savings by almost 40 percent when taxes are set optimally compared to the initial economy. Higher deposit savings increases capital used in firm's production by a similar magnitude. In contrast, the real housing stock falls by almost 25 percent. The reduction in the nominal housing stock is even larger. As house prices fall more than 16 percent, the nominal value of the housing stock is down 36 percent. The substantial reallocation of capital increases output by close to 10 percent.


Figure 1: Optimal taxation in steady state
Note: Figure 1a shows welfare changes in percent for newborns across different property tax levels. Welfare in terms of efficiency is computed according to equation (25) with $\hat{\sigma}=0$. Utilitarian welfare is computed according to equation (25) with $\hat{\sigma}=\sigma$. The current property tax level in the U.S. is one percent. Figure 1b shows the capital income tax rate needed to keep government expenditures $G$ constant across different property tax levels.

|  |  |  |  | Initial economy | Optimal steady state |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Normalized variables |  |  |  |  |  |
| $D:$ | Deposits |  |  |  |  |
| $K:$ | Capital | 1 |  |  |  |
| $H:$ | Real housing stock | 1 |  |  |  |
| $p_{h} H:$ | Nominal housing stock | 1 |  |  |  |
| $Y:$ | Output | 1.385 |  |  |  |
| $p_{h}:$ | House price | 1 |  |  |  |
| $p_{r}: \quad$ Rental price | 1 | 0.707 |  |  |  |
| $C: \quad$ Consumption | 1 | 0.640 |  |  |  |
| $w: \quad$ Wage level | 1 | 0.095 |  |  |  |
| Other variables | 1 | 1.117 |  |  |  |
| $r: \quad$ Interest rate (\%) |  | 1.065 |  |  |  |
| $\bar{r}: \quad$ Interest rate after tax (\%) | 6.60 | 1.095 |  |  |  |
| Price-to-rent ratio | 4.22 |  |  |  |  |
| Fraction homeowners | 9.14 | 3.93 |  |  |  |
| Fraction homeowners, below age 35 | 0.68 | 4.16 |  |  |  |

Table 3: Change in key aggregate variables: initial versus optimal utilitarian steady state Note: In the initial steady state, the property tax is one percent and the capital income tax rate is 36 percent. The optimal utilitarian property tax rate is 7.1 percent and the optimal capital income tax rate is -5.7 percent.

Households' welfare is negatively affected by lower housing consumption and a fall in the overall homeownership rate. The net benefit of owning is reduced as owner-occupied
housing does not receive a preferential tax treatment in the optimal steady state. As a result, the average homeownership rate goes down by almost one third to 44 percent. The drop is especially large for younger households as they postpone their housing purchases. The price of rental units is up by almost 12 percent as the financial intermediary must be compensated for a higher property tax. Thus, the demand for rental housing also falls.

The negative effects of a higher property tax are outweighed by a higher wage level and an increase in non-housing consumption. As a result of higher capital accumulation, the wage level is up by close to 10 percent. Higher earnings lead to an increase in non-housing consumption of almost 7 percent. Higher earnings also help dampening the fall in housing consumption.

In contrast to the theoretical models in Section 2, the optimal property tax varies across households. The theoretical models suggest that all households want a capital income tax of zero and thus the same property tax (see also Appendix A. 2 for a brief discussion). Figure 2 shows that this does not hold in the quantitative exercise. The figure shows that the optimal property tax rate decreases with initial labor productivity $n_{i 1}{ }^{20}$ Whereas the welfare of households at the bottom 20 percent of the distribution is maximized when the property tax is 8 percent, the top 20 percent are best off when the property tax rate is 5 percent. Poorer households benefit greatly from higher wages. Richer households also gain from higher wages but are less eager to raise the property tax. They also reap benefits from the current tax system which allows them to reduce their overall tax burden by investing in owner-occupied housing. Overall, these results help explain why a utilitarian planner, who assigns a higher weight to poorer households, wants a slightly higher property tax rate compared to a planner who only cares about efficiency.

How important are endogenous house prices for the results? Qualitatively, the results are similar in a model with perfectly elastic housing and a constant house price (see Table 4 and Appendix Table D.1): the optimal property tax is substantially higher and the capital income tax is close to zero; households benefit from a reallocation of capital, which increases wages; and the fraction of homeowners fall. Quantitatively, there are some differences. First, the optimal property tax rate is lower in a model with a constant house price (see Table 4). With endogenous house prices, house prices fall and the increase in rental prices is more muted following an increase in property taxes. As a result, the real housing stock falls less compared to a model with fixed prices. Yet, the nominal housing value drops considerably more. To finance a lower capital income tax, a higher property tax is required. Second, the welfare gains are larger in a model with endogenous

[^13]

Figure 2: Optimal property tax rates (\%) across initial labor productivity
Note: Initial labor productivity $n_{i 1}$ is the productivity of household $i$ at age $j=1$. Households are divided into quintiles based on their productivity and each marker shows the property tax rate which maximizes average welfare within a specific quintile.
house prices. Lower house prices allow households to keep up the consumption of housing services to a greater extent. Lower housing costs also leave more room to save in deposits, which has a positive impact on the wage level.

|  | Benchmark | Constant $p_{h}$ |
| :--- | :---: | :---: |
| Efficiency |  |  |
| Optimal $\tau^{h}(\%)$ | 6.1 | 4.2 |
| Optimal $\tau^{k}(\%)$ | -0.8 | 0.7 |
| Welfare change (\%) | 3.4 | 2.7 |
| Utilitarian |  |  |
| Optimal $\tau^{h}(\%)$ | 7.1 | 5.0 |
| Optimal $\tau^{k}(\%)$ | -5.7 | -5.9 |
| Welfare change (\%) | 4.4 | 3.6 |

Table 4: Optimal steady-state taxes
Note: In the benchmark model, house prices are endogenous. $\tau^{h}$ is the property tax rate, $\tau^{k}$ is the capital income tax, and $p_{h}$ is the house price. Optimal taxes in terms of efficiency are found by maximizing equation (25) with $\hat{\sigma}=0$. Optimal utilitarian taxes are found by maximizing equation (25) with $\hat{\sigma}=\sigma$.

## 6 Optimal taxation with transitional dynamics

In this section, I carefully consider the transitional dynamics following a tax reform. Ex ante, it is not clear that the optimal property tax should be considerably higher than today after including these transitional effects. First, wages do not jump immediately
to new steady-state levels after a tax reform. It takes time to increase the capital stock. Second, sudden changes to the tax system may not be appreciated by all of the current generations. For example, a large increase in property tax payments is likely to negatively affect homeowners. Third, both business capital and housing capital are considerably less elastic in the short run. Thus, in terms of efficiency, it may be good to tax both types of capital more heavily.

### 6.1 Welfare measure and planner problem

The welfare measure needs to capture the complexity of the problem at hand. Importantly, the problem is no longer confined to maximizing the welfare of newborns in the long run. The welfare measure also needs to consider the impact on current generations and the newborn generations that enter the economy along the long-lived transition. To capture the consequences for all of these generations, the social welfare function takes the following form

$$
\begin{equation*}
\left(\sum_{g=1}^{J} \Lambda_{g 1} \int_{0}^{1} \omega_{i g 1}\left(\boldsymbol{\tau}^{h}\right)^{1-\hat{\sigma}} \mathrm{d} i+\sum_{t=2}^{\infty} \Lambda_{1 t} \int_{0}^{1} \omega_{i 1 t}\left(\boldsymbol{\tau}^{h}\right)^{1-\hat{\sigma}} \mathrm{d} i\right)^{\frac{1}{1-\hat{\sigma}}} \tag{26}
\end{equation*}
$$

where the first term captures the welfare of the generations alive at the time of a policy change and the second term captures the welfare of households that enter the economy later in the transition. $\omega_{i g t}\left(\boldsymbol{\tau}^{h}\right)$ is the constant per-period consumption equivalent for household $i$ in the $g^{\prime}$ th generation at time $t$ under a certain policy $\boldsymbol{\tau}^{h} \equiv\left\{\tau_{t}^{h}\right\}_{t=1}^{\infty}$. In Appendix B.2, I provide a detailed account of how the individual consumption equivalents are computed. $\Lambda_{g t}$ is the weight assigned to the $g^{\prime}$ th generation at time $t$. For example, $\Lambda_{21}$ is the weight assigned to the second generation alive in the first period of the transition and $\Lambda_{12}$ is the weight assigned to a newborn generation in the second period of the transition. At any time $t$, the weight is higher for younger generations as they constitute a larger share of the population and because they expect to live longer. The weight assigned to each generation also depends on family size through a equivalence scale $e_{g}$. Moreover, a social discount factor $\Theta^{t-1}$ decides the weight of current generations relative to newborn generations at time $t$. The higher is the social discount factor, the higher is the weight on the welfare of future generations. I will set $\Theta$ equal to the the private discount factor $\beta$. A formal definition of $\Lambda_{g t}$ is relegated to Appendix B.2. Again, $\hat{\sigma}$ captures the inequality aversion of the planner.

As in Section 5, I analyze optimal policies using two welfare measures. The first welfare measure considers aggregate efficiency and implies $\hat{\sigma}=0$. The second welfare measure represents utilitarian welfare by assuming that the planner's inequality aversion
equals households' risk aversion $\sigma$. In Appendix B. 2, I show how utilitarian welfare can be represented by expression (26) when $\hat{\sigma}=\sigma$.

The planner problem is to choose a policy $\tau^{h}$, i.e., a sequence of the property tax rate $\left\{\tau_{t}^{h}\right\}_{t=1}^{\infty}$ to maximize expression (26) subject to a value of $\hat{\sigma}$ and a number of constraints. In the main analysis, I consider once-and-for-all type of policies, which means that the property tax changes immediately to the new long-run level, i.e., $\left\{\tau_{t}^{h}\right\}_{t=1}^{\infty}=\tau_{\text {new }}^{h} \forall t$. The policies are assumed to be credible and unexpected. ${ }^{21}$ The permanent one-time change in the property tax rate will give rise to a sequence of the capital income tax rate $\left\{\tau_{t}^{k}\right\}_{t=1}^{\infty}$, a sequence of the bequest parameter $\left\{\gamma_{t}\right\}_{t=1}^{\infty}$, a sequence of the interest rate $\left\{r_{t}\right\}_{t=1}^{\infty}$, and a sequence of the house price $\left\{p_{h, t}\right\}_{t=1}^{\infty}$. The capital income tax rate ensures that the tax revenues exactly cover government expenditures $G$. The bequest parameter ensures that bequests received equal bequests left behind. The interest rate clears the capital asset market. Finally, the house price is set to clear the housing market. I also provide results of a model with fixed house prices.

### 6.2 Main results

The main result of the quantitative analysis is that the optimal property tax is significantly higher than today even after considering the transitional dynamics. How much higher the optimal property tax is compared to status quo depends on the social welfare function (see Figure 3). A utilitarian planner wants to increase the property tax to 4.8 percent, which is more than a planner who cares solely about efficiency. But even in terms of efficiency, the optimal property tax is close to three times higher than the current level of one percent. Recall that the corresponding optimal taxes were 7.1 percent and 6.1 percent, respectively, in the steady-state analysis. Thus, the increase in property taxes is somewhat lower when transitional dynamics are accounted for.

The optimal capital income tax is lower than today but positive in the long run. Table 5 shows that the optimal capital income tax for a utilitarian planner and a planner who only cares about efficiency is 6.2 percent and 22.1 percent, respectively, in the long run. Both are significantly lower than the current capital income tax of 36 percent. However, it takes time before the capital income tax reaches its long-run level. For example, consider the dynamics of the capital income tax after implementing the optimal utilitarian policy. ${ }^{22}$

[^14]

Figure 3: Welfare effects (\%) with transitional dynamics
Note: Welfare in terms of efficiency is computed according to equation (26) with $\hat{\sigma}=0$. Utilitarian welfare is computed according to equation (26) with $\hat{\sigma}=\sigma$. The current property tax level in the U.S. is one percent.

At first, the capital income tax is actually lower than its long-run level. As the capital stock is predetermined at the time of the reform, the government does not need to tax the capital income stock at high levels to ensure tax neutrality.In the periods that follow, the capital income tax is higher than the long-run level and decreases only slowly towards the new steady-state level. The capital income tax is higher than its long-run level as low house prices reduce the revenue from property taxation for an extended period.

The welfare effects are more dispersed when the long-lived transitional dynamics are included. This helps explain why the quantitative differences in optimal policies of a planner who maximizes efficiency and a utilitarian planner are larger compared to the steady-state analysis. Broadly speaking, a utilitarian planner wants a higher property tax as newborns and relatively poor renters among the current generations benefit considerably from higher taxes. In what follows, I provide a more detailed account of how a sudden change to the property tax affects households' welfare.

The average welfare gain of a newborn generation is positive but lower for newborns that enter the economy shortly after an increase in the property tax relative to those who enter later. Figure 4 shows the average welfare effects of implementing the optimal utilitarian policy for newborn generations across time. It shows that the welfare increase is significant for all newborn generations. Also the first generation of newborns after the reform benefits from higher wages and experiences an average increase in consumption equivalents of more than one percent. However, it takes time for the wage to increase to its long-run level. Thus, the welfare gain among newborns in the long run is substantially higher. On average, their welfare increases by almost four percent.


Figure 4: Welfare change (\%) for newborn generations under the optimal utilitarian policy Note: The welfare effect at each point in time corresponds to the average welfare effect of the newborn generation that enters the economy in that specific period. The optimal utilitarian policy refers to the policy which maximizes equation (25) with $\hat{\sigma}=\sigma$. The policy is implemented unexpectedly in period one (year 3).

The optimal property tax for current generations is lower than today's level of one percent. How much lower the optimal tax is, depends on the social welfare function. For example, Figure 5a shows that the property tax that maximizes welfare in terms of efficiency is as low as zero percent, whereas utilitarian welfare is maximized at a somewhat higher level of 0.4 percent. On average, current generations incur large welfare losses from implementing the policies that maximize the welfare of both current and future generations. However, these average consequences mask important heterogeneous effects.

Across age groups, retirees lose the most from a higher property tax (see Figure 5b). As an example, assume that the property tax changes to 4.8 percent, which is the optimal property tax for a utilitarian planner who also values future generations. Then, the average welfare loss among current retirees is equivalent to a consumption fall of almost six percent. In contrast, the average welfare gain among retirees from decreasing the property tax to zero is equivalent to an increase in consumption of around two percent. Retirees tend to own their homes, are less affected by changes in the wage, have started to eat off their deposit savings, and have fewer periods left to live. Thus, they are negatively affected by an increase in the property tax payments and they gain less from lower capital income taxes and higher wages. Working-age households, excluding newborns, are on average also better off with lower property taxes. However, the welfare gain of setting the property tax to zero is lower than for retirees and the welfare loss of implementing the optimal utilitarian policy is more muted.

On average, current homeowners are considerably better off with a lower property tax (see Figure 5c). Homeowners incur welfare losses from higher property taxes as property


Figure 5: Welfare consequences for current generations
Note: Figure 5b shows the average welfare change in percent for households alive today, where households are divided into three age groups. "Newborns" constitutes ages $23-25$, "Other working-age households" covers the ages $26-64$, whereas "Retirees" includes the remainder. Similarly, Figure 5 c shows the average welfare change based on the housing situation of a household prior to the policy change. "Smaller owned houses" refers to households that own $\underline{h}$, whereas "Larger owned houses" refers to households that own houses of a size larger than $\underline{h}$. A household is assumed to be in favor of a policy if its welfare effect is greater than or equal to zero.
tax payments increase and falling house prices reduce their net wealth and the value of bequests. Moreover, large transaction costs lock homeowners into house sizes that are no longer optimal in their view. The negative consequences are larger for households who own smaller homes. In my model, there is a strong positive correlation between housing and deposits. Thus, households with larger homes also appreciate a reduction in capital income taxes as these households tend to have larger deposit holdings. In contrast to
homeowners, renters are relatively poor households that benefit from higher wages and lower house and rental prices. Moreover, they can freely adjust how much housing services they consume.

As most households own their home, a majority of current households are against an increase in the property tax rate. Figure 5d shows the fraction in favor of each policy reform. A household is assumed to be in favor of a policy if its welfare effect is greater than or equal to zero. The figure shows that less than one third of current households are in favor of increasing the property tax. In stark contrast, around two thirds of all households, and almost all homeowners, are in favor of reducing the property tax to zero.

|  | Current generations $(\Theta=0)$ <br> Benchmark |  | All generations $(\Theta=\beta)$ |  |
| :--- | :---: | :---: | :---: | :---: |
| Constant $p_{h}$ | Benchmark | Constant $p_{h}$ |  |  |

Table 5: Optimal taxes with transitional dynamics
Note: In the benchmark model, house prices are endogenous. $\tau^{h}$ is the property tax rate, $\tau^{k}$ is the long-run capital income tax, $p_{h}$ is the house price, $\Theta$ is the social discount factor, and $\beta$ is the private discount factor. Optimal taxes in terms of efficiency are found by maximizing equation (26) with $\hat{\sigma}=0$. Optimal utilitarian taxes are found by maximizing equation (26) with $\hat{\sigma}=\sigma$. A household is assumed to be in favor of a policy if its welfare effect is greater than or equal to zero.

A model with constant house prices would miss important distributional effects among current generations. In fact, Table 5 shows that maximizing utilitarian welfare for current generations without allowing for endogenous house prices entails more than a doubling of the current property tax rate. In particular, the welfare losses of current homeowners are considerably lower when the house price is fixed (see Appendix Figure E.3c). Notwithstanding these results, the optimal property taxes for a planner who cares about all generations are qualitatively, and to a great extent quantitatively, similar in a model with constant house prices (see also Appendix Figure E.2).

### 6.3 Time-varying policies

The once-and-for-all policies suggest that even if a planner wants to increase property taxes, it may be politically difficult to do so. A fundamental issue is that future generations
benefit from higher property taxes, whereas current generations on average are better off with a lower property tax. This raises the question of whether there exists any policy that can make current households better off and allows for higher property taxes in the long run. Here, I will present one potentially fruitful policy. Specifically, I consider a time-varying policy which first lowers the property tax rate to compensate current homeowners before the tax rate increases to the benefit of future generations. ${ }^{23}$

I find that the time-varying policies can improve the welfare of a majority of current households. Figure 6a shows the average welfare change among current generations, where I vary the number of years the property tax is set to zero before it increases to the optimal utilitarian level of 4.8 percent found in Section 6.2. The figure shows that current generations may benefit from such a policy. For example, average welfare in terms of efficiency is positive if the property tax is kept at zero percent for 60 years. More than 60 percent of current households would be in favor of this policy (see Figure 6b). Clearly, also newborn generations in the long run will benefit from all of the policies, as the property tax eventually increases to the higher long-run value.

It is crucial that the property tax is low for a long period. A lower property tax also involves costs: to ensure tax neutrality, the capital income tax increases; and there is less investments in business capital, which lower wages. Moreover, if the property tax increases too quickly, house prices may continue to fall in the short run or house prices do not increase enough to compensate homeowners. Figure 6b shows that the property tax needs to be zero for at least 45 years to ensure that the benefits outweigh the costs for a majority of current households. For example, if the property tax is only lowered for a 15 year-period, less than 20 percent of current households are better off. In this latter case, a higher share of households prefer the once-and-for-all policy.

On the downside, the time-varying policies negatively impact many newborn generations along the transition. In Figures 6 c and 6 d , I analyze the welfare impact on newborn generations of two policies. In the first policy, the property tax is never zero and immediately jumps to the long-run level. This is similar to the benchmark results in Section 6.2. As previously discussed, most newborn households are in favor of this policy. In the second policy, I consider a policy where the property tax is zero for 60 years before it increases to the long-run level. This reform results in negative welfare effects for many of the newborn generations that enter the economy after the policy is implemented. The welfare losses are substantial. Indeed, Figure 6e shows that the welfare measure of a social planner who cares about current and future households is negative for all of the policies

[^15]

Figure 6: Optimal taxes with time-varying policies
Note: The welfare effect at each point in time corresponds to the average welfare effect of the newborn generation that enters the economy in that specific period. Similarly, the fraction of newborns in favor of a policy at each point in time refers to the newborn generation that enters the economy in that specific period.
that a majority of current households benefit from.

## 7 Robustness

In this section, I briefly discuss the sensitivity of my results to changing some of the key assumptions I have made in the analysis thus far.

### 7.1 Steady state

In Appendix D.2, I show that the steady-state results are qualitatively, and to a large extent quantitatively, robust to a range of alternative modeling assumptions. ${ }^{24}$ First, I increase the intertemporal elasticity of substitution by assuming log utility (i.e., $\sigma=1$ ). This increases the cost of taxing capital and reduces the desire for the planner to redistribute across households. Second, I reduce the transaction costs of buying and selling owneroccupied housing by half. Third, I disregard the utility of leaving bequests. It is not obvious how to best model a bequest motive, so it is reassuring that the results hold even when there is no bequest utility. However, if I do not impose tax neutrality and thus let government expenditures change across policies, the optimal property tax is zero. This aligns well with the theoretical results in Section 2, where key benefits come from reducing the capital income tax. Still, one should be careful putting too much weight on this result as the negative impact of reduced government spending is not included in this exercise.

### 7.2 Main results

The results from the once-and-for-all policies, which include transitional dynamics, are also qualitatively, and in many cases quantitatively, robust to changing several key assumptions and parameters. The results from the sensitivity analyses are shown in Appendix Table D.2. The table shows optimal policies for current generations as well as for the overall welfare of households that includes future generations.

A higher intertemporal elasticity of substitution reduces the desire for a utilitarian planner to redistribute.As poorer households benefit the most from higher property taxes, the optimal utilitarian property tax is lower with log utility as compared to the benchmark case with $\sigma=2$. The optimal property tax is zero percent for a utilitarian planner who only considers the welfare of current generations, whereas it was 0.4 percent when households' risk aversion was 2 . The optimal property tax for a utilitarian planner who values the welfare of all generations continues to be substantially higher than today, albeit at a somewhat lower level. Specifically, the optimal property tax is 3.5 percent with the

[^16]assumption of log utility. Recall that the benchmark model implied an optimal property tax of 4.8 percent. The optimal capital income tax is 17 percent, which is around ten percentage points higher than in the benchmark model. The optimal policies with log utility for a planner who only cares about efficiency are quantitatively similar to those in the benchmark model.

The results are quantitatively insensitive to changing the transaction costs of housing and abstracting from a bequest motive. Specifically, I: i) reduce the transaction cost of buying and selling owner-occupied housing by half; ii) remove the bequest motive altogether; and iii) abstract from the bequest motive when computing social welfare. ${ }^{25}$

I also test if it is important for aggregate welfare that the capital gain $\Delta p_{h, t}$ from increasing house prices leads to lower rental prices (see equation (7)). The assumption that higher house prices over the transition lower rental prices may be unrealistic and overestimate the welfare gains among poorer households. As an alternative, I assume that the capital gain is fully taxed by the government and used to fund government expenditures. With this alternative assumption, a utilitarian planner who only cares about current generations would optimally choose to set the property tax to zero. However, the optimal taxes for a planner who considers all generations remain quantitatively very similar to the benchmark model.

As in the steady-state analysis, the assumption of tax neutrality is important. If there is no change in the capital income tax, the optimal property tax is zero. Again, this analysis disregards the negative effects of reduced government revenues and must be interpreted with some caution.

### 7.3 Time-varying policies

The time-varying results do not depend on the exact level of the long-run property tax rate. In Section 6.3, I vary the number of years the property tax is zero, but always assume that the property tax increases to the optimal utilitarian level of 4.8 percent. However, one may argue that it should be easier for current generations to accept a policy if the increase in the long-run property tax is less substantial. Thus, I re-run the exercise in Table 6, where the long-run property tax is only half as high, i.e., 2.4 percent. Overall, the results remain very similar.

The time-varying results are also robust to assuming that the property tax increases exponentially to the long-run level. In Section 6.3, I assume that the property tax is zero for a certain time, before it jumps to its long-run level. An interesting alternative is to consider the effects of a smoother transition. Thus, I consider a case where the property

[^17]tax is reduced to a level just above zero before it grows exponentially to its long-run level within a certain time period. The results of this exercise are also very similar to the results in Figure 6. ${ }^{26}$

## 8 Concluding remarks

In this paper, I consider a new way of thinking about residential property taxation that acknowledges the important role of housing for the economy at large. Taking a consolidated government budget approach, I study how a property tax interacts with a tax on capital income and how various combinations of the two affect economic efficiency and households' welfare. To better understand who wins and who loses from different property tax policies, I complement a traditional representative agent framework with a quantitative analysis using a heterogeneous agent model.

A main finding in this paper is that standard welfare measures imply an optimal property tax rate that is substantially higher than today. This result holds both in steady state and after including the transitional dynamics following a reform. Moreover, the quantitative results in this paper are robust to, e.g., holding house prices fixed, doubling the intertemporal elasticity of substitution, and halving the transactions costs of housing.

Still, the welfare effects are widely dispersed across households and generations, which makes the political economy of introducing higher property taxes challenging. While future generations and current renters typically benefit from a higher property tax, current homeowners incur large losses on average. As most households own their home, it may prove difficult to implement a higher tax. On the positive side, I show that time-varying policies can make a majority of current households in favor of a substantial future increase in the property tax rate. But even these policies are somewhat unsatisfactory, as they leave many newborn generations worse off compared to the status quo tax system. Overall, my results open the floor for an interesting discussion of exactly how property taxes can be increased.

Future research can usefully expand my analysis along several dimensions. First, this paper takes an important initial step by considering how the government as a whole should finance its expenditures. An interesting next step is to decide which government level that is best suited for taxing the income and property of households. Second, the planner has no incentive to extract land rent by increasing property taxes in the current framework. As land is typically in relatively fixed supply, taxation of land rent is close to non-distortionary. Thus, the present analysis misses a potentially important argument for higher property taxes. However, in this regard, I want to highlight a noteworthy

[^18]contribution of this paper: namely, I show that there are good reasons to tax housing more than today even if the supply of housing is perfectly elastic. Third, it would be fruitful to study optimal taxes that also allow for some progressivity in the tax rates. Finally, to concentrate on the interactions between housing and other capital, I assume that labor is inelastically supplied by households. A natural way forward would be to relax this assumption and consider the effects of including endogenous labor supply. While all these extensions deserve a serious investigation, they also add further complexity to an already challenging problem and I leave them for future research to explore.

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## Appendices (For Online Publication)

## A Property taxation in the Chamley (1986) model

## A. 1 Representative agent

To see how housing and property taxation may affect the original Chamley-Judd result, I study a modified version of the Chamley model presented in Atkeson et al. (1999). ${ }^{27}$ Assume that there is a representative household that lives forever and has the discount factor $\beta<1$. The expected discounted utility is given by

$$
\begin{equation*}
\sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}, s_{t}\right), \tag{27}
\end{equation*}
$$

where $c_{t}$ is non-housing consumption and $s_{t}$ is housing services in period $t$. Households maximize (27) subject to a budget constraint

$$
\begin{equation*}
\sum_{t=0}^{\infty} p_{t}\left(c_{t}+\left(1+\tau_{t}^{h}\right) s_{t}+k_{t+1}\right)=\sum_{t=0}^{\infty} p_{t}\left(1+\bar{r}_{t}\right) k_{t} \tag{28}
\end{equation*}
$$

with $k_{0}>0$. The price level in period $t$ is $p_{t}$, and I normalize prices in period zero to 1 . For simplicity, I assume that the housing supply is perfectly elastic, which implies that the pre-tax price of $c_{t}$ and $s_{t}$ is the same. The property tax in period $t$ is $\tau_{t}^{h}$, whereas $k_{t}$ is capital at the start of the period and $k_{t+1}$ are savings in capital from period $t$ to period $t+1$. The after-tax return on savings in capital is $\bar{r}_{t}=\left(1-\tau_{t}^{k}\right)\left(R_{t}-\delta^{k}\right)$, where $\tau_{t}^{k}$ is the capital income tax at time $t, R_{t}$ is the rental price of capital at time $t$, and $\delta^{k}$ is the depreciation rate of capital.

The household chooses $c_{t}, s_{t}$, and $k_{t+1}$ and the first-order conditions of the household problem are

$$
\begin{align*}
\beta^{t} U_{c t} & =\lambda p_{t}  \tag{29}\\
\beta^{t} U_{s t} & =\lambda p_{t}\left(1+\tau_{t}^{h}\right)  \tag{30}\\
\lambda p_{t} & =\lambda p_{t+1}\left(1+\bar{r}_{t+1}\right), \tag{31}
\end{align*}
$$

where $\lambda$ is the Lagrange multiplier on the budget constraint (28), and $U_{c t}$ and $U_{s t}$ are the derivatives of the utility function with respect to $c_{t}$ and $s_{t}$, respectively. The household's

[^19]Euler equation is derived by substituting (29) into (31):

$$
\begin{equation*}
U_{c t}=\beta\left(1+\bar{r}_{t+1}\right) U_{c, t+1} . \tag{32}
\end{equation*}
$$

A representative firm produces output goods $y_{t}=F\left(k_{t}\right)$ using capital $k_{t}$ as input. The firm's maximization problem is

$$
\begin{equation*}
\max _{k_{t}} F\left(k_{t}\right)-R_{t} k_{t} . \tag{33}
\end{equation*}
$$

The firm's first-order condition with respect to capital implies

$$
\begin{equation*}
F_{k t}=R_{t}, \tag{34}
\end{equation*}
$$

where $F_{k t}$ is the derivative of $F\left(k_{t}\right)$ with respect to $k_{t}$. The government's revenues from taxing housing and capital income are spent on government expenditures $g$, which are assumed to be fixed throughout. Formally, the government budget constraint is

$$
\sum_{t=0}^{\infty} p_{t} g=\sum_{t=0}^{\infty} p_{t}\left(\tau_{t}^{h} s_{t}+\tau_{t}^{k}\left(R_{t}-\delta^{k}\right) k_{t}\right) .
$$

Finally, there is a resource constraint which must hold every period

$$
c_{t}+s_{t}+k_{t+1}+g=F\left(k_{t}\right)+\left(1-\delta^{k}\right) k_{t}
$$

A competitive equilibrium can be fully characterized by the resource constraint and an implementability constraint. The implementability constraint is the household budget constraint (28), where I substitute for the first-order conditions of the household (29)-(31) and the first-order condition of the firm (34), i.e.,

$$
\sum_{t=0}^{\infty} \beta^{t}\left(U_{c t} c_{t}+U_{s t} s_{t}\right)=U_{c 0}\left(1+\bar{r}_{0}\right) k_{0},
$$

where $\bar{r}_{0}=\left(1-\tau_{0}^{k}\right)\left(F_{k 0}-\delta^{k}\right)$ and $\tau_{0}^{k}$ is given.
Assume that the government can perfectly commit to any sequence of tax rates $\tau_{t}^{h}$ and $\tau_{t}^{k}$ for all time periods $t$. Assume further that the government needs to choose policies that are compatible with a competitive equilibrium. Denote the social welfare function in period $t$ as

$$
W\left(c_{t}, s_{t}, \mu\right)=U\left(c_{t}, s_{t}\right)+\mu\left(U_{c t} c_{t}+U_{s t} s_{t}\right),
$$

where $\mu$ is the multiplier on the implementability constraint. Then the Ramsey allocation
problem can be summarized as follows

$$
\begin{aligned}
\max _{c_{t}, s_{t}, k_{t+1}} & \sum_{t=0}^{\infty} \beta^{t} W\left(c_{t}, s_{t}, \mu\right)-\mu U_{c 0}\left(1+\bar{r}_{0}\right) k_{0} \\
& \quad+\chi_{t}\left(F\left(k_{t}\right)+\left(1-\delta^{k}\right) k_{t}-c_{t}-s_{t}-g-k_{t+1}\right),
\end{aligned}
$$

where $\chi_{t}$ is the multiplier on the resource constraint in period $t$. The first-order conditions yield

$$
\begin{array}{ll}
\frac{W_{c t}}{W_{s t}}=1 & \\
W_{c t}=\beta W_{c, t+1}\left(F_{k, t+1}+1-\delta^{k}\right) & \text { for } t \geq 1 \\
W_{c 0}=\beta W_{c 1}\left(F_{k 1}+1-\delta^{k}\right)+\mu U_{c c 0}\left(1+\bar{r}_{0}\right) k_{0} & \text { for } t=0 \tag{37}
\end{array}
$$

Equation (36) is key to the Chamley (1986) result on zero capital income taxation in the long run. Since the Ramsey equilibrium is a competitive equilibrium, both the intertemporal condition for the planner (36) and the Euler equation (32) must hold. Suppose that the economy reaches a steady state with $\left(c_{t}, s_{t}, k_{t}\right)=(c, s, k)$ for all $t$, such that $W_{c t}=W_{c, t+1}$ and $U_{c t}=U_{c, t+1}$. It then follows that the Euler equation (32) is equal to the intertemporal optimality condition of the planner (36) if and only if $\left(1-\tau_{t}^{k}\right)\left(F_{k, t+1}-\delta^{k}\right)$ is equal to $F_{k, t+1}-\delta^{k}$. In turn, this implies that $\tau^{k}$ has to be zero in steady state. Overall, the addition of housing and a property tax rate does not change the key finding of the Chamley model.

## A. 2 Heterogeneous agents

Atkeson et al. (1999) show that the basic Chamley result is robust to allowing for households with heterogeneous capital endowments. The important insight is that in a model with heterogeneous agents, there is one Euler equation for each agent and a corresponding intertemporal condition for the planner for each agent. Thus, the planner finds it optimal to set the capital income tax rates to zero. This also has the stark implication that independently of the weight placed on different agents, the optimal capital income tax is zero. Although their result is based on the trade-off between taxing labor versus capital, it is relatively easy to see, given the previous analysis, that the result holds if housing rather than labor is taxed.

## A. 3 A two-period OLG model

Assume that at each time $t$, there is one young generation (indexed 1) and one old generation (indexed 2) alive. The young generation wants to maximize its discounted utility over the two periods

$$
\begin{equation*}
U^{1}\left(c_{1 t}, s_{1 t}, n_{1 t}\right)+\beta U^{2}\left(c_{2, t+1}, s_{2, t+1}\right), \tag{38}
\end{equation*}
$$

where $c_{1 t}, s_{1 t}$ and $n_{1 t}$ are non-housing consumption, consumption of housing services, and efficiency hours worked when young, respectively, $\beta$ is the discount factor, and $c_{2, t+1}$ and $s_{2, t+1}$ are non-housing consumption and consumption of housing services when old.

The budget constraint for the young is

$$
\begin{equation*}
c_{1 t}+\left(1+\tau_{1 t}^{h}\right) s_{1 t}+k_{t+1}+b_{t+1}=w_{t} n_{1 t} \tag{39}
\end{equation*}
$$

and for the old it is

$$
\begin{equation*}
c_{2, t+1}+\left(1+\tau_{2, t+1}^{h}\right) s_{2, t+1}=\left(1+\bar{r}_{t+1}\right)\left(k_{t+1}+b_{t+1}\right), \tag{40}
\end{equation*}
$$

where $\tau_{1 t}^{h}$ is the property tax for young households in period $t, \tau_{2, t+1}^{h}$ is the property tax for old households in period $t+1, \bar{r}_{t+1}=\left(1-\tau_{t+1}^{k}\right)\left(R_{t+1}-\delta^{k}\right)$ is the after-tax return on savings, $b_{t+1}$ is government debt held by the young generation, and the young generation has no initial capital as there is no bequest motive. I allow for type-specific tax rates to simplify the analysis. The household chooses $c_{1 t}, s_{1 t}, n_{1 t}, k_{t+1}, b_{t+1}, c_{2 t+1}$, and $s_{2 t+1}$. The corresponding first-order conditions are

$$
\begin{align*}
U_{c 1 t}^{1} & =\lambda_{1 t}  \tag{41}\\
U_{s 1 t}^{1} & =\lambda_{1 t}\left(1+\tau_{1 t}^{h}\right)  \tag{42}\\
U_{n 1 t}^{1} & =-\lambda_{1 t} w_{t}  \tag{43}\\
\lambda_{1 t} & =\lambda_{2, t+1}\left(1+\bar{r}_{t+1}\right),  \tag{44}\\
\lambda_{1 t} & =\lambda_{2, t+1}\left(1+\bar{r}_{t+1}\right),  \tag{45}\\
\beta U_{c 2, t+1}^{2} & =\lambda_{2, t+1}  \tag{46}\\
\beta U_{s 2, t+1}^{2} & =\lambda_{2, t+1}\left(1+\tau_{2, t+1}^{h}\right) \tag{47}
\end{align*}
$$

where $\lambda_{1 t}$ is the Lagrange multiplier on the budget constraint for the young, and $\lambda_{2, t+1}$ is the Lagrange multiplier on the budget constraint for the old. Note that these multipliers
are not necessarily constant across time. The Euler equation is as follows

$$
\begin{equation*}
U_{c 1 t}^{1}=\beta U_{c 2, t+1}^{2}\left(1+\bar{r}_{t+1}\right) . \tag{48}
\end{equation*}
$$

The firm problem is

$$
\begin{equation*}
\max _{k_{t}, n_{1 t}} F\left(k_{t}, n_{1 t}\right)-R_{t} k_{t}-w_{t} n_{1 t} . \tag{49}
\end{equation*}
$$

The rental rate and the wage level are given by the firm's first-order conditions of capital and labor

$$
\begin{align*}
R_{t} & =F_{k t}  \tag{50}\\
w_{t} & =F_{n 1 t} . \tag{51}
\end{align*}
$$

The government constraint is

$$
\begin{equation*}
g+\bar{r}_{t} b_{t}=\tau_{1 t}^{h} s_{1 t}+\tau_{2 t}^{h} s_{2 t}+\tau_{t}^{k}\left(R_{t}-\delta^{k}\right) k_{t}+b_{t+1} \tag{52}
\end{equation*}
$$

The resource constraint is given by

$$
\begin{equation*}
c_{1 t}+c_{2 t}+s_{1 t}+s_{2 t}+k_{t+1}+g=F\left(k_{t}, n_{1 t}\right)+\left(1-\delta^{k}\right) k_{t}, \tag{53}
\end{equation*}
$$

where $k_{0}>0$. The derivation of the implementability constraint (IC) is slightly different in this case. Start by substituting in for the first-order conditions for $c_{1 t}(41), s_{1 t}$ (42), and $n_{1 t}$ (43) in the budget constraint for the young generation (39)

$$
\begin{equation*}
U_{c 1 t}^{1} c_{1 t}+U_{s 1 t}^{1} s_{1 t}+U_{n 1 t}^{1} n_{1 t}=-\lambda_{1 t}\left(k_{t+1}+b_{t+1}\right) \tag{54}
\end{equation*}
$$

Continue by substituting in for the first-order conditions for $c_{2, t+1}(46)$ and $s_{2, t+1}(47)$ in the budget constraint for the old (40)

$$
\begin{align*}
& \beta\left(U_{c 2, t+1}^{2} c_{2, t+1}+U_{s 2, t+1}^{2} s_{2, t+1}\right)=\lambda_{2 t+1}\left(1+\bar{r}_{t+1}\right)\left(k_{t+1}+b_{t+1}\right) \\
& \beta\left(U_{c 2, t+1}^{2} c_{2, t+1}+U_{s 2, t+1}^{2} s_{2, t+1}\right)=\lambda_{1 t}\left(k_{t+1}+b_{t+1}\right), \tag{55}
\end{align*}
$$

where I used equation (44) to get from the first to the second equation. Setting (54) equal to (55), I get the implementability constraint

$$
\begin{equation*}
U_{c 1 t}^{1} c_{1 t}+U_{s 1 t}^{1} s_{1 t}+U_{n 1 t}^{1} n_{1 t}=-\beta\left(U_{c 2, t+1}^{2} c_{2, t+1}+U_{s 2, t+1}^{2} s_{2, t+1}\right) . \tag{56}
\end{equation*}
$$

Note that the resource constraint (53) and the implementability constraint represented by (56) constitute a competitive equilibrium. Denote the social welfare function in period $t$

$$
\begin{align*}
W\left(c_{1 t}, s_{1 t}, n_{1 t}, c_{2, t+1}, s_{2, t+1}, \mu_{t}\right) & =U^{1}\left(c_{1 t}, s_{1 t}, n_{1 t}\right)  \tag{57}\\
& +\beta U^{2}\left(c_{2, t+1}, s_{2, t+1}\right) \\
& +\mu_{t}\left[U_{c 1 t}^{1} c_{1 t}+U_{s 1 t}^{1} s_{1 t}+U_{n 1 t}^{1} n_{1 t}\right. \\
& \left.+\beta\left(U_{c 2, t+1}^{2} c_{2, t+1}+U_{s 2, t+1}^{2} s_{2, t+1}\right)\right],
\end{align*}
$$

where $\mu_{t}$ is the multiplier on the implementability constraint. In this model, the Ramsey planner needs to assign a weight $\Theta^{t}$ with $\Theta<1$ to agents in generation $t$. Specifically, the planner wants to maximize

$$
\begin{equation*}
\max \frac{U^{2}\left(c_{20}\right)}{\Theta}+\sum_{t=0}^{\infty} \Theta^{t} W\left(c_{1 t}, s_{1 t}, n_{1 t}, c_{2, t+1}, s_{2, t+1}, \mu_{t}\right) \tag{58}
\end{equation*}
$$

where the utility of the current old is given by $U^{2}\left(c_{20}\right)$. The planner maximizes (58) subject to the resource constraint (53). The optimality conditions are

$$
\begin{align*}
\Theta^{t} W_{c 1 t} & =\Theta^{t} \chi_{t}  \tag{59}\\
\Theta^{t} W_{s 1 t} & =\Theta^{t} \chi_{t}  \tag{60}\\
\Theta^{t} W_{n 1 t} & =-\Theta^{t} \chi_{t} F_{n 1 t}  \tag{61}\\
\Theta^{t} W_{c 2 t} & =\Theta^{t+1} \chi_{t+1}  \tag{62}\\
\Theta^{t} W_{s 2 t} & =\Theta^{t+1} \chi_{t+1}  \tag{63}\\
\Theta^{t} \chi_{t} & =\Theta^{t+1} \chi_{t+1}\left(F_{k, t+1}+1-\delta^{k}\right) \tag{64}
\end{align*}
$$

where $\Theta^{t} \chi_{t}$ is the multiplier on the resource constraint in period $t$. Rearrange the optimality conditions to get

$$
\begin{equation*}
W_{c 1 t}=\Theta W_{c 1, t+1}\left(F_{k, t+1}+1-\delta^{k}\right) \tag{65}
\end{equation*}
$$

Assume that the economy converges to a steady state such that $\left(c_{1 t}, s_{1 t}, n_{1 t}, c_{2 t}, s_{2 t}, k_{t+1}\right)=$ $\left(c_{1}, s_{1}, n_{1}, c_{2}, s_{2}, k\right)$ for all $t$. Then I can rewrite (65) as

$$
\begin{equation*}
\Theta^{-1}=F_{k}+1-\delta^{k} . \tag{66}
\end{equation*}
$$

From the Euler equation (48), I have

$$
\begin{equation*}
\frac{U_{c 1}^{1}}{\beta U_{c 2}^{2}}=1+\left(1-\tau^{k}\right)\left(F_{k}-\delta^{k}\right) \tag{67}
\end{equation*}
$$

Comparing (66) and (67) we see that the capital income tax in steady state for this economy is zero only if

$$
\begin{equation*}
\Theta^{-1}=\frac{U_{c 1}^{1}}{\beta U_{c 2}^{2}} \tag{68}
\end{equation*}
$$

The next step is to see when this holds. Use the first-order condition for $c_{1 t}$ (59) and $c_{2, t+1}(62)$ to arrive at the following expression in steady state

$$
\begin{align*}
\Theta & =\frac{W_{c 2}}{W_{c 1}} \\
& =\frac{\frac{W_{c 2}}{U_{c 2}^{2}} U_{c 2}^{2}}{\frac{W_{c 1}}{U_{c 1}} U_{c 1}^{1}} . \tag{69}
\end{align*}
$$

It is relatively easy to see that if $\frac{W_{c 2}}{U_{c 2}^{2}} / \frac{W_{c 1}}{U_{c 1}^{1}}=\beta$, then the capital income tax is zero in steady state. I now show that this holds for the following utility functions

$$
\begin{align*}
U^{1}\left(c_{1}, s_{1}, n_{1}\right) & =\left(c_{1}^{\alpha} s_{1}^{1-\alpha}\right)^{1-\sigma} /(1-\sigma)+V\left(n_{1}\right)  \tag{70}\\
U^{2}\left(c_{2}, s_{2}\right) & =\left(c_{2}^{\alpha} s_{2}^{1-\alpha}\right)^{1-\sigma} /(1-\sigma) . \tag{71}
\end{align*}
$$

Derive $W_{c 1} / U_{c 1}^{1}$

$$
\begin{align*}
& W_{c 1}=U_{c 1}^{1}+\mu_{t}\left[U_{c c 1}^{1} c_{1}+U_{c 1}^{1}+U_{s c 1}^{1} s_{1}+U_{n c 1}^{1} n_{1}\right] \\
& \frac{W_{c 1}}{U_{c 1}^{1}}=1+\mu_{t}[1-\sigma] . \tag{72}
\end{align*}
$$

Derive $W_{c 2} / U_{c 2}^{2}$

$$
\begin{align*}
W_{c 2} & =\beta\left(U_{c 2}^{2}+\mu_{t}\left[U_{c c 2}^{2} c_{2}+U_{c 2}^{2}+U_{s c 2}^{2} s_{2}\right]\right) \\
\frac{W_{c 2}}{U_{c 2}^{2}} & =\beta\left(1+\mu_{t}[1-\sigma]\right) . \tag{73}
\end{align*}
$$

Together (69), (72), and (73) imply $\Theta^{-1}=U_{c 1}^{1} /\left(\beta U_{c 2}^{2}\right)$. Thus, in steady state, the capital income tax is zero.

## B Welfare measures

## B. 1 Utilitarian welfare in steady state

Here, I want to show that utilitarian welfare is indeed maximized if (25) in section 5 is maximized and $\hat{\sigma}=\sigma$. First, note that utilitarian welfare is given by the sum of individual welfare $\int_{0}^{1} V_{i}\left(\tau^{h}\right) \mathrm{d} i$. In turn, any level of the utilitarian welfare can be represented by a consumption equivalent measure. This is done by solving for a consumption equivalent that is common to all households $\bar{\omega}\left(\tau^{h}\right)$ such that

$$
\begin{equation*}
\int_{0}^{1} V_{i}\left(\tau^{h}\right) \mathrm{d} i=\int_{0}^{1} \tilde{\beta}^{j} \frac{\bar{\omega}\left(\tau^{h}\right)^{1-\sigma}}{1-\sigma} \mathrm{d} i \tag{74}
\end{equation*}
$$

where $\tilde{\beta}^{j} \equiv e_{j} \beta^{j-1} \prod_{k=1}^{j-1} \phi_{k}$ is the effective discount factor for consumption at age $j$ from the perspective of a newborn.

The next step is to see that individual welfare $V_{i}\left(\tau^{h}\right)$ can be substituted by individual consumption equivalents $\omega_{i}\left(\tau^{h}\right)$ as in equation (24) in section 5 . Substituting equation (24) into equation (74) gives

$$
\int_{0}^{1} \tilde{\beta}^{j} \frac{\omega_{i}\left(\tau^{h}\right)^{1-\sigma}}{1-\sigma} \mathrm{d} i=\int_{0}^{1} \tilde{\beta}^{j} \frac{\bar{\omega}\left(\tau^{h}\right)^{1-\sigma}}{1-\sigma} \mathrm{d} i .
$$

It is possible to solve for $\bar{\omega}\left(\tau^{h}\right)$ as it does not depend on $i$ and $j$. This renders

$$
\bar{\omega}\left(\tau^{h}\right)=\left(\int_{0}^{1} \omega_{i}\left(\tau^{h}\right)^{1-\sigma} \mathrm{d} i\right)^{\frac{1}{1-\sigma}}
$$

where the right-hand side of the equation is exactly the measure that is assumed to capture utilitarian welfare in (25).

## B. 2 Welfare when considering transitional dynamics

Let total utility, including the warm-glow bequest motive, for household $i$ of age $j$ at time $t$ be

$$
W_{i j t}\left(\boldsymbol{\tau}^{h}\right) \equiv U_{j}\left(c_{i j t}\left(\boldsymbol{\tau}^{h}\right), s_{i j t}\left(\boldsymbol{\tau}^{h}\right)\right)+\beta\left(1-\phi_{j}\right) U^{B}\left(q_{i j t}^{\prime}\left(\boldsymbol{\tau}^{h}\right)\right),
$$

where $c_{i j t}, s_{i j t}$, and $q_{i j t}^{\prime}$ are realized values of consumption, housing services and net worth, and $\boldsymbol{\tau}^{h}$ is a specific property tax policy $\left\{\tau_{t}^{h}\right\}_{t=1}^{\infty}$.

The value function for household $i$ of the $g^{\prime}$ th generation at time $t$ under a policy $\boldsymbol{\tau}^{h}$ is

$$
V_{i g t}\left(\boldsymbol{\tau}^{h}\right) \equiv \sum_{j=g}^{J}\left(\beta^{j-g}\left(1 / \phi_{g}\right) \prod_{k=g}^{j} \phi_{k}\right) W_{i j t}\left(\boldsymbol{\tau}^{h}\right),
$$

where $t=j-g+1$ is the time period for the utility flow, and $\beta^{j-g}\left(1 / \phi_{g}\right) \prod_{k=g}^{j} \phi_{k}$ is the effective discount factor for streams of utility at age $j$ for a household of generation $g$. For example, when $g=j$, we are in the first period of the transition so there should be no discounting of utility, i.e., $\beta^{0}=1$ and $\left(1 / \phi_{g}\right) \prod_{k=g}^{j} \phi_{k}=\left(\phi_{g} / \phi_{g}\right)=1$.

Next, let $\omega_{i g t}\left(\tau^{h}\right)$ capture welfare in consumption terms for household $i$ of the $g^{\prime}$ th generation at time $t$ under policy $\boldsymbol{\tau}^{h}$. Specifically, $\omega_{i g t}\left(\boldsymbol{\tau}^{h}\right)$ solves

$$
\begin{equation*}
V_{i g t}\left(\boldsymbol{\tau}^{h}\right)=\sum_{j=g}^{J} \tilde{\beta}^{j} \frac{\omega_{i g t}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma}}{1-\sigma} \tag{75}
\end{equation*}
$$

where households discount the consumption stream with factor $\tilde{\beta}^{j} \equiv e_{j} \beta^{j-g} \frac{1}{\phi_{g}} \prod_{k=g}^{j} \phi_{k}$.
Before I formally show the weight assigned to different households, denote the normalized population distribution by $\tilde{\Pi}_{g} \equiv \Pi_{g} / \Pi_{1}$, such that each generation of newborns is normalized to one. Then, the weight assigned to the $g^{\prime}$ th generation at time $t$ is given by

$$
\begin{equation*}
\Lambda_{g t}=\frac{\Theta^{t-1} \tilde{\Pi}_{g} \sum_{j=g}^{J} \tilde{\beta}^{j}}{\sum_{j=1}^{J} \tilde{\Pi}_{j} \sum_{k=j}^{J} \tilde{\beta}^{k}+\sum_{t=2}^{\infty} \Theta^{t-1} \sum_{j=1}^{J} \tilde{\beta}^{j}}, \tag{76}
\end{equation*}
$$

where $\Theta \in[0,1[$ is the social discount factor. The numerator in equation (76) can be explained as follows. Younger generations receive a higher weight as they constitute a larger share of the population $\left(\tilde{\Pi}_{g}\right)$ and because they expect to live longer $\left(\sum_{j=g}^{J} \tilde{\beta}^{j}\right)$. $\Theta^{t-1}$ simply states that, all else equal, a social planner cares more about the welfare of current generations than the welfare of future generations. The longer into the future the welfare accrues, the lower is the weight assigned to the newborn generation. Loosely speaking, one can think of the numerator as representing the number of discounted "effective" years of the $g^{\prime}$ 'th generation at time $t$ from the perspective of a social planner. Similarly, the denominator can be thought of as the total number of discounted "effective" years for all generations across time. The first term in the denominator captures the "effective" years among the current generations. Notice that there is no $\Theta$ in the first term because there is no need to discount the welfare of current generations. The second term in the denominator represents the "effective" years among all future newborn generations, which is a finite number as $\Theta<1$. There is no $\tilde{\Pi}_{1}$ in the second term because of the normalization of the population distribution.

Now that I have defined individual welfare $\omega_{i g t}\left(\boldsymbol{\tau}^{h}\right)$ and the weight assigned to the
$g^{\prime}$ th generation at time $t$, it is time to show that utilitarian welfare is maximizized if (26) in section 6 is maximized and $\hat{\sigma}=\sigma$. Similar to Appendix B.1, solve for the consumption equivalent that is common to all households $\bar{\omega}\left(\boldsymbol{\tau}^{h}\right)$ such that

$$
\begin{equation*}
\sum_{g=1}^{J} \Lambda_{g 1} \int_{0}^{1} \frac{\omega_{i g 1}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma}}{1-\sigma} \mathrm{d} i+\sum_{t=2}^{\infty} \Lambda_{1 t} \int_{0}^{1} \frac{\omega_{i 1 t}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma}}{1-\sigma} \mathrm{d} i=\frac{\bar{\omega}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma}}{1-\sigma} \tag{77}
\end{equation*}
$$

where the left-hand side of the equation represents utilitarian welfare in terms of individual consumption equivalents. The right-hand side of the equation does not feature any weights or integrals as the consumption equivalent $\bar{\omega}\left(\boldsymbol{\tau}^{h}\right)$ is common to all generations at any time $t$. With some minor algebra, $\bar{\omega}\left(\boldsymbol{\tau}^{h}\right)$ is given by

$$
\bar{\omega}\left(\boldsymbol{\tau}^{h}\right)=\left(\sum_{g=1}^{J} \Lambda_{g 1} \int_{0}^{1} \omega_{i g 1}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma} \mathrm{d} i+\sum_{t=2}^{\infty} \Lambda_{1 t} \int_{0}^{1} \omega_{i 1 t}\left(\boldsymbol{\tau}^{h}\right)^{1-\sigma} \mathrm{d} i\right)^{\frac{1}{1-\sigma}}
$$

where the right-hand side of the equation is the function that is assumed to represent utilitarian welfare in (26).

## C Equilibrium definitions

Households are heterogeneous with respect to age $j \in \mathcal{J} \equiv\{1,2, \ldots, J\}$, labor productivity $n \in \mathcal{N} \equiv \mathbb{R}_{++}$, cash-on-hand $x \in \mathcal{X} \equiv \mathbb{R}_{++}$, owner-occupied housing $h \in \mathcal{H} \equiv$ $\{0, \underline{\mathrm{~h}}, \ldots, \bar{h}=\bar{s}\}$, and mortgage $m \in \mathcal{M} \equiv \mathbb{R}_{+}$. Let $\mathcal{Z} \equiv \mathcal{N} \times \mathcal{X} \times \mathcal{H} \times \mathcal{M}$ be the non-deterministic state space with $\mathbf{z} \equiv(n, x, h, m)$ denoting the vector of individual states. Let $\mathbf{B}\left(\mathbb{R}_{++}\right)$and $\mathbf{B}\left(\mathbb{R}_{+}\right)$be the Borel $\sigma$-algebras on $\mathbb{R}_{++}$and $\mathbb{R}_{+}$respectively, and $P(\mathcal{H})$ the power set of $\mathcal{H}$, and define $\mathscr{B}(\mathcal{Z}) \equiv \mathbf{B}\left(\mathbb{R}_{++}\right) \times \mathbf{B}\left(\mathbb{R}_{++}\right) \times P(\mathcal{H}) \times \mathbf{B}\left(\mathbb{R}_{+}\right)$. Further, let $\mathbb{M}$ be the set of all finite measures over the measurable space $(\mathcal{Z}, \mathscr{B}(\mathcal{Z}))$. Then $\Phi_{j t} \in \mathbb{M}$ is a probability measure defined on subsets $Z \in \mathscr{B}(\mathcal{Z})$ that describes the distribution of individual states across agents with age $j \in \mathcal{J}$ at time $t$. Finally, denote the time-invariant fraction of the population of age $j \in \mathcal{J}$ by $\Pi_{j}$.

Definition 1. Given a sequence of property tax rates $\left\{\tau_{t}^{h}\right\}_{t=1}^{t=\infty}$, government expenditures $G$, available land $L$, and initial conditions $\Phi_{j 1}$ for all $j$, a recursive competitive equilibrium with partly inelastic housing supply is a sequence of value functions $\left\{V_{j t}(\mathbf{z})\right\}_{t=1}^{t=\infty}$ with associated policy functions $\left\{c_{j t}(\mathbf{z}), s_{j t}(\mathbf{z}), h_{j t}^{\prime}(\mathbf{z}), m_{j t}^{\prime}(\mathbf{z}), d_{j t}^{\prime}(\mathbf{z})\right\}_{t=1}^{t=\infty}$ for all $j$; a sequence of prices $\left\{\left(p_{h, t}, p_{r, t}, r_{t}, w_{t}\right)\right\}_{t=1}^{t=\infty}$; a social security $\operatorname{tax} \tau^{s s}$; a sequence of bequest parameters $\left\{\gamma_{t}\right\}_{t=1}^{t=\infty}$; a sequence of capital income taxes $\left\{\tau_{t}^{k}\right\}_{t=1}^{t=\infty}$; a sequence of production plans for the production firm $\left\{N, K_{t},\right\}_{t=1}^{t=\infty}$; a sequence of rental stocks $\left\{H_{f, t}\right\}_{t=1}^{t=\infty}$; a sequence of housing stocks $\left\{H_{t}\right\}_{t=1}^{t=\infty}$; a sequence of investment plans for the construction
firm $\left\{I_{h, t+1}\right\}_{t=1}^{t=\infty}$; a sequence of land prices $\left\{\tau_{t}^{L}\right\}_{t=1}^{t=\infty}$; and a sequence of distributions of agents' states $\left\{\Phi_{j t}\right\}_{t=1}^{t=\infty}$ for all $j$ such that:

1. Given prices $\left(p_{h, t}, p_{r, t}, w_{t}, r_{t}\right)$ and parameters $\left(t_{t}^{k}, \tau^{s s}, \gamma_{t}\right), V_{j t}(\mathbf{z})$ solves the Bellman equation (4) with the corresponding set of policy functions for all $j$ and $t$ : $\left\{c_{j t}(\mathbf{z}), s_{j t}(\mathbf{z}), h_{j t}^{\prime}(\mathbf{z}), m_{j t}^{\prime}(\mathbf{z}), d_{j t}^{\prime}(\mathbf{z})\right\}$.
2. The interest rate $r_{t}$ and the wage level $w_{t}$ satisfy (5) and (6), respectively.
3. The rental price $p_{r, t}$ satisfies the financial intermediary's optimality condition (7).
4. The investment plan $I_{h, t}$ is given by (8).
5. The housing stock $H_{t}$ satisfies the law of motion for aggregate housing (9).
6. The land price $\tau_{t}^{L}$ ensures that the construction firm makes zero profits.
7. The payroll $\operatorname{tax} \tau^{s s}$ satisfies (10).
8. The bequest parameter $\gamma_{t}$ balances bequests left and bequests received (12).
9. The capital income tax $\tau_{t}^{k}$ balances the government budget (13).
10. The aggregate resource constraint (14) holds, where

- aggregate consumption $C_{t}$ is given by (15);
- aggregate transaction costs $\Omega_{t}$ are given by (17);
- aggregate labor supply $N$ satisfies (18);
- and aggregate capital $K_{t}$ is given by (19).

11. The capital market satisfies (20).
12. The rental stock $H_{f, t}$ satisfies (22).
13. Distributions of states $\Phi_{j t}$ are given by the following law of motion for all $j<J$ and $t$ :

$$
\Phi_{j+1, t+1}(\mathcal{Z})=\int_{Z_{t}} T_{j t}(\mathbf{z}, \mathcal{Z}) d \Phi_{j t}\left(Z_{t}\right)
$$

where $T_{j t}: \mathcal{Z} \times \mathscr{B}(\mathcal{Z}) \rightarrow[0,1]$ is a transition function that defines the probability that a household of age $j$ at time $t$ transits from its current state $\mathbf{z}$ to the set $\mathcal{Z}$ at age $j+1$ and time $t+1$.
Definition 2. A stationary equilibrium is a competitive equilibrium in which all tax policies, value functions, policy functions, prices and other market-clearing parameters, as well as aggregate quantities, are constant.

## D Additional steady-state results

## D. 1 Optimal steady-state taxation with constant house prices

|  | Initial economy | Optimal steady state |  |
| :---: | :---: | :---: | :---: |
|  |  | Benchmark | Constant $p_{h}$ |
| Normalized variables |  |  |  |
| D: Deposits | 1 | 1.385 | 1.368 |
| K: Capital | 1 | 1.407 | 1.372 |
| $H$ : Real housing stock | 1 | 0.765 | 0.747 |
| $p_{h} H$ : Nominal housing stock | 1 | 0.640 | 0.747 |
| $Y$ : Output | 1 | 1.095 | 1.087 |
| $p_{h}$ : House price | 1 | 0.837 | 1 |
| $p_{r}$ : $\quad$ Rental price | 1 | 1.117 | 1.170 |
| $C$ : Consumption | 1 | 1.065 | 1.074 |
| $w$ : Wage level | 1 | 1.095 | 1.087 |
| Other variables |  |  |  |
| $r: \quad$ Interest rate (\%) | 6.60 | 3.93 | 4.11 |
| $\bar{r}$ : Interest rate after tax (\%) | 4.22 | 4.16 | 4.36 |
| Price-to-rent ratio | 9.14 | 6.85 | 7.82 |
| Fraction homeowners | 0.68 | 0.44 | 0.44 |
| Fraction homeowners, below age 35 | 0.35 | 0.18 | 0.17 |

Table D.1: Change in key aggregate variables in steady state with constant house prices Note: In the initial steady state, the property tax is one percent and the capital income tax rate is 36 percent. Optimal taxes maximize utilitarian welfare (see Table 4 for optimal values). In the benchmark model, house prices are endogenous.


Figure D.1: Optimal taxation in steady state with constant house prices
Note: Figure D.1a shows welfare changes in percent for newborns across different property tax levels. Welfare in terms of efficiency is computed according to equation (25) with $\hat{\sigma}=0$. Utilitarian welfare is computed according to equation (25) with $\hat{\sigma}=\sigma$. The current property tax level in the U.S. is one percent. Figure D.1b shows the capital income tax rate needed to keep government expenditures $G$ constant across different property tax levels.


Figure D.2: Optimal property tax rates (\%) with constant house prices across initial labor productivity
Note: Initial labor productivity $n_{i 1}$ is the productivity of household $i$ at age $j=1$. Households are divided into quintiles based on their productivity and each marker shows the property tax rate which maximizes average welfare within a specific quintile.

## D. 2 Further sensitivity analyses

|  | Optimal $\tau^{h}$ <br> $(\%)$ | Optimal $\tau^{k}$ <br> $(\%)$ | Welfare change <br> $(\%)$ | Fraction <br> in favor |
| :--- | :---: | :---: | :---: | :---: |
| Efficiency |  |  |  |  |
| Benchmark | 6.1 | -0.8 | 3.4 | 0.97 |
| Constant $p_{h}$ | 4.2 | 0.7 | 2.7 | 0.96 |
| Higher IES (log utility) | 6.0 | 0.7 | 3.0 | 0.97 |
| Halved transaction costs housing | 5.9 | 1.8 | 3.5 | 0.98 |
| No bequest motive | 6.5 | -4.8 | 3.5 | 0.97 |
| No bequest utility in social welfare measure | 6.1 | -0.8 | 3.5 | 0.97 |
| No tax neutrality | 0 | N.A. | 0.7 | 0.90 |
| No tax neutrality or balanced bequests | 0 | N.A. | 0.3 | 0.62 |
| Utilitarian |  |  |  |  |
| Benchmark |  |  |  |  |
| Constant $p_{h}$ |  | -5.7 | 4.4 | 0.97 |
| Higher IES (log utility) | 5.0 | -5.9 | 3.6 | 0.96 |
| Halved transaction costs housing | 6.0 | 0.7 | 3.4 | 0.97 |
| No bequest motive | 6.6 | -1.4 | 4.4 | 0.97 |
| No bequest utility in social welfare measure | 7 | -7.2 | 4.6 | 0.97 |
| No tax neutrality | 7.1 | -5.7 | 4.4 | 0.97 |
| No tax neutrality or balanced bequests | 0 | N.A. | 0.6 | 0.90 |

Table D.2: Sensitivity analyses of optimal steady-state taxes
Note: In the benchmark model, house prices are endogenous. $\tau^{h}$ is the property tax rate, $\tau^{k}$ is the capital income tax, and $p_{h}$ is the house price. Optimal taxes in terms of efficiency are found by maximizing equation (25) with $\hat{\sigma}=0$. Optimal utilitarian taxes are found by maximizing equation (25) with $\hat{\sigma}=\sigma$.

## E Additional results with transitional dynamics

## E. 1 Dynamics of key parameters and variables



Figure E.1: Dynamics of key parameters and variables under the optimal utilitarian policy Note: The optimal utilitarian policy refers to the policy which maximizes equation (25) with $\hat{\sigma}=\sigma$.

## E. 2 Optimal taxation with constant house prices



Figure E.2: Welfare effects (\%) with transitional dynamics and constant house prices Note: Welfare in terms of efficiency is computed according to equation (26) with $\hat{\sigma}=0$. Utilitarian welfare is computed according to equation (26) with $\hat{\sigma}=\sigma$. The current property tax level in the U.S. is one percent.


Figure E.3: Constant house prices and the welfare consequences for current generations
Note: Figure 5b shows the average welfare change in percent for households alive today, where households are divided into three age groups. "Newborns" constitutes ages $23-25$, "Other working-age households" covers the ages $26-64$, whereas "Retirees" includes the remainder. Similarly, Figure 5c shows the average welfare change based on the housing situation of a household prior to the policy change. "Smaller owned houses" refers to households that own $\underline{h}$, whereas "Larger owned houses" refers to households that own houses of a size larger than $\underline{h}$. A household is assumed to be in favor of a policy if its welfare effect is greater than or equal to zero.

## E. 3 Further sensitivity analyses

|  | Optimal $\tau^{h}$ <br> (\%) | Optimal $\tau^{k}$ <br> (\%) | Welfare change (\%) | Fraction in favor |
| :---: | :---: | :---: | :---: | :---: |
| Current generations ( $\Theta=0$ ) |  |  |  |  |
| Efficiency |  |  |  |  |
| Benchmark | 0 | 46.8 | 0.5 | 0.63 |
| Constant $p_{h}$ | 0.8 | 38.1 | 0.01 | 0.51 |
| Higher IES (log utility) | 0 | 42.0 | 0.3 | 0.64 |
| Halved transaction costs housing | 0 | 45.4 | 0.6 | 0.64 |
| No bequest motive among households | 0 | 46.5 | 0.5 | 0.64 |
| No bequest utility in social welfare measure | 0 | 46.8 | 0.6 | 0.63 |
| $p_{r}$ unaffected by capital gain $\Delta p_{h}$ | 0 | 46.8 | 0.5 | 0.63 |
| No tax neutrality | 0 | N.A. | 2.3 | 0.996 |
| No tax neutrality or balanced bequests | 0 | N.A. | 2.0 | 0.98 |
| Utilitarian |  |  |  |  |
| Benchmark | 0.4 | 41.9 | 0.1 | 0.65 |
| Constant $p_{h}$ | 2.3 | 20.7 | 0.2 | 0.41 |
| Higher IES (log utility) | 0 | 42.0 | 0.3 | 0.64 |
| Halved transaction costs housing | 0.4 | 41.9 | 0.1 | 0.65 |
| No bequest motive among households | 0.6 | 39.9 | 0.03 | 0.66 |
| No bequest utility in social welfare measure | 0.4 | 42.2 | 0.04 | 0.65 |
| $p_{r}$ unaffected by capital gain $\Delta p_{h}$ | 0 | 46.8 | 0.1 | 0.63 |
| No tax neutrality | 0 | N.A. | 2.1 | 0.996 |
| No tax neutrality or balanced bequests | 0 | N.A. | 1.7 | 0.98 |
| All generations ( $\Theta=\beta$ ) |  |  |  |  |
| Efficiency |  |  |  |  |
| Benchmark | 2.6 | 22.1 | 0.2 | 0.33 |
| Constant $p_{h}$ | 2.4 | 19.5 | 0.4 | 0.40 |
| Higher IES (log utility) | 2.2 | 26.9 | 0.1 | 0.34 |
| Halved transaction costs housing | 2.4 | 24.1 | 0.2 | 0.35 |
| No bequest motive among households | 2.5 | 22.6 | 0.7 | 0.33 |
| No bequest utility in social welfare measure | 2.4 | 23.7 | 0.1 | 0.33 |
| $p_{r}$ unaffected by capital gain $\Delta p_{h}$ | 2.5 | 22.9 | 0.2 | 0.32 |
| No tax neutrality | 0 | N.A. | 1.7 | 0.996 |
| No tax neutrality or balanced bequests | 0 | N.A. | 1.3 | 0.98 |
| Utilitarian |  |  |  |  |
| Benchmark | 4.8 | 6.2 | 1.1 | 0.33 |
| Constant $p_{h}$ | 3.9 | 4.9 | 1.4 | 0.34 |
| Higher IES (log utility) | 3.5 | 17.0 | 0.4 | 0.34 |
| Halved transaction costs housing | 5.3 | 4.8 | 1.2 | 0.35 |
| No bequest motive among households | 5.2 | 2.8 | 1.4 | 0.32 |
| No bequest utility in social welfare measure | 4.8 | 6.2 | 1.2 | 0.33 |
| $p_{r}$ unaffected by capital gain $\Delta p_{h}$ | 4.7 | 6.7 | 0.9 | 0.31 |
| No tax neutrality | 0 | N.A. | 1.4 | 0.996 |
| No tax neutrality or balanced bequests | 0 | N.A. | 0.9 | 0.98 |

Table E.1: Sensitivity analyses of optimal taxes with transitional dynamics
Note: In the benchmark model, house prices are endogenous. $\tau^{h}$ is the property tax rate, $\tau^{k}$ is the long-run capital income tax, $p_{h}$ is the house price, $\Theta$ is the social discount factor, and $\beta$ is the private discount factor. Optimal taxes in terms of efficiency are found by maximizing equation (26) with $\hat{\sigma}=0$. Optimal utilitarian taxes are found by maximizing equation (26) with $\hat{\sigma}=\sigma$. A household is assumed to be in favor of a policy if its welfare effect is greater than or equal to zero.


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[^1]:    ${ }^{1}$ Another way to nullify the preferential tax-treatment of housing is to tax the imputed rent of owner-occupied housing (see, e.g., Gervais (2002)). While this is an interesting alternative to consider, taxes on imputed rents are rarely seen in reality.

[^2]:    ${ }^{2}$ All policies are assumed to be credible and implemented unexpectedly. Similar once-and-for-all policies are assumed in, e.g., Domeij and Heathcote (2004), Bakış et al. (2015), and Krueger and Ludwig (2016) who study optimal capital taxation, optimal progressivity of the income distribution, and the optimal provision of social insurance, respectively.
    ${ }^{3}$ I show that a model with exogenous house prices severely underestimates the costs to current homeowners.

[^3]:    ${ }^{4}$ A list of key papers on the more general topic of optimal taxation includes, but is not limited to, Summers (1981), Auerbach et al. (1983), Judd (1985), Chamley (1986), Aiyagari (1995), İmrohoroğlu (1998), Atkeson et al. (1999), Domeij and Heathcote (2004), Conesa et al. (2009), and Straub and Werning (2020).

[^4]:    ${ }^{5}$ Straub and Werning (2020) show that this result does not necessarily follow from the models in Chamley (1986) and Judd (1985). For example, the Judd (1985) model would entail positive capital income taxation in the long run if the intertemporal elasticity of substitution is below or equal to one. The critique against the Chamley (1986) model is less strong, and Straub and Werning (2020) show that with additively separable utility, the zero-tax result also applies to models with an intertemporal elasticity below or equal to one. However, for this to hold, the upper bounds on capital income taxes must be slack in the long run.
    ${ }^{6}$ Straub and Werning (2020) show that this argument does not hold if the initial government debt is sufficiently large. When government expenditures are high, it is beneficial to tax capital income to alleviate the efficiency costs of taxing labor income. With upper bounds on capital income taxes, it may be optimal to tax capital even in the long run.
    ${ }^{7}$ Importantly, this version of the Chamley model is not subject to the critique in Straub and Werning (2020) as it assumes an additively separable utility function and bounds on the capital income tax are only imposed in the first period.

[^5]:    ${ }^{8}$ Alternatively, consider a household that chooses between becoming a landlord or to invest in financial assets. In the U.S., landlords must tax the income from providing rental services, but can deduct expenses, such as property taxes, before paying taxes. Thus, the return of being a landlord is $\left(1-\tau^{k}\right)\left(p_{r}-\tau^{h}\right)$, where $\tau^{k}$ is the capital income tax. The return from investing in financial assets is $\left(1-\tau^{k}\right) r$. Thus, a no-arbitrage condition implies $p_{r}=r+\tau^{h}$.

[^6]:    ${ }^{9}$ Notice that the property tax is not a tax on the imputed rent as any change in the property tax affects the rental cost and the flow cost for owner-occupied housing to an equal extent.

[^7]:    ${ }^{10}$ Primes indicate the current period choice of variables that affect next period's state variables.
    ${ }^{11}$ The main results in this paper also hold if I do not adjust $\bar{q}$ with the wage level.
    ${ }^{12}$ It is thus convenient to require homeowners to pay for maintenance, as the house size could otherwise effectively end up between the specified discrete values.

[^8]:    ${ }^{13}$ A minimum size of owner-occupied housing $\underline{h}$ is also assumed in, e.g., Cho and Francis (2011), Floetotto et al. (2016), Gervais (2002), and Sommer and Sullivan (2018).

[^9]:    ${ }^{14}$ The law-of-motion for the housing stock can be used to find the fixed value of new land $L$. Without loss of generality, I set the house price $p_{h, t+1}$ to one in the initial steady state. Since $p_{h, t}=p_{h, t+1}=1$ and $H_{t}=H_{t+1}=H$ in steady state, $L$ is equal to $\delta^{h} H$, i.e., the new land covers the depreciated housing stock.
    ${ }^{15}$ Favilukis et al. (2017) make a similar assumption.
    ${ }^{16}$ Whereas the theoretical models discussed in Appendix A, either implicitly or explicitly, include government debt, my quantitative framework does not feature government debt. In this regard, my model assumption is similar to that in Conesa et al. (2009). As noted by these authors, the optimal capital income tax need not be zero in steady state when government debt is disregarded. It can be negative as well as positive.

[^10]:    ${ }^{17}$ I use the survey years 1989 to 2013 for the SCF, where all waves are pooled.

[^11]:    ${ }^{18}$ See table C-10-OO in the 2013 American Housing Survey.

[^12]:    ${ }^{19}$ The computational method to solve the model is similar to the one in Karlman et al. (2021).

[^13]:    ${ }^{20}$ Based on their initial productivity, households are divided into quintiles and each marker shows the property tax rate that maximizes average individual welfare within a specified quintile.

[^14]:    ${ }^{21}$ As the policy change is unexpected, I adjust households' cash-on-hand in the first period of the transition in three ways. First, the amount of bequests received fall as house prices drop. Second, cash-on-hand is adjusted for the new property tax rate and the capital income tax rate. Third, the profits of the rental business and the construction firm become negative due to the unexpected change in the property tax. The profit loss is taken lump-sum from households.
    ${ }^{22}$ See the dynamics of key parameters and variables, including the capital income tax, in Appendix Figure E.1.

[^15]:    ${ }^{23}$ In this analysis, I make a small, but important, change to the LTV constraint. Specifically, I assume that the LTV requirement will be based on the house price in the next period whenever the agents know that the house price will fall. Without this assumption, households will end up with very high LTVs in the period that the property tax increases to its long-run level.

[^16]:    ${ }^{24}$ Whenever necessary, I recalibrate the model.

[^17]:    ${ }^{25}$ When I disregard the bequest motive, I keep the social discount factor $\Theta$ at the benchmark level.

[^18]:    ${ }^{26}$ The results from these two robustness tests are available upon request.

[^19]:    ${ }^{27}$ I abstract from labor income, except when I discuss results in an overlapping-generations model. This simplification is not as stark as it may first appear. In fact, all results still hold if I allow for an endogenous labor supply and an additively-separable disutility of labor.

