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Abstract

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JEL No. C12, C21, R15

Keywords: Double Length Regression; Spatial Lag Dependence; Spatial Error Dependence; Artificial Regressions; Panel Data; Fixed Effects

We dedicate this paper in honor of Aman Ullah's many contributions to econometrics. We would like to thank two anonymous referees for their helpful comments and suggestions.

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Testing For Spatial Lag and Spatial Error Dependence in a Fixed Effects Panel Data Model Using Double Length Artificial Regressions*

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August 26, 2015

Abstract

This paper revisits the joint and conditional Lagrange Multiplier tests derived by Debarsy and Ertur (2010) for a fixed effects spatial lag regression model with spatial auto-regressive error, and derives these tests using artificial Double Length Regressions (DLR). These DLR tests and their corresponding LM tests are compared using an empirical example and a Monte Carlo simulation.

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JEL Classification: C12, C21, R15.

1 Introduction

Davidson and MacKinnon (1984, 1988, 1993) proposed Double Length Regressions (DLR) as a useful tool for deriving LM tests. These DLR's are computationally simple and outperform their outer product gradient (OPG) regression counterparts. They have been applied in econometrics by Baltagi (1999) to test linear versus log-linear regressions with AR(1) disturbances. Also, Baltagi and Li (2001) to test for spatial dependence in a cross-section regression model, and by Le and Li (2008) who applied it to test for functional form and spatial error dependence, to mention a few. Recently, Baltagi and Liu (2014) extended the Baltagi and Li (2001) paper by deriving the DLR tests corresponding to the joint and conditional LM tests of spatial lag and spatial error in a cross-section regression model. Testing for spatial dependence in a cross-section regression has been extensively studied by Anselin (1988a, 1988b, 2001), Anselin and Bera (1998), Anselin, Bera, Florax and Yoon (1996), and Krämer (2005), to mention a few. The presence of spatial correlation can

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render inference using ordinary least squares misleading, see Krämer (2003). Also, Mynbaev and Ullah (2008) who derive the asymptotic distribution of the OLS estimator in a spatial autoregressive model.[1] Anselin et al. (1996) consider a spatial autoregressive cross-section regression model with spatial autoregressive disturbances and derive a series of Lagrange Multiplier (LM) tests. These are called the joint, conditional and marginal LM tests for spatial lag and spatial error dependence. In fact, Baltagi and Li (2001) derived the DLR counterpart for the marginal LM tests for spatial lag and spatial error considered by Anselin et al. (1996) and illustrated these tests using Anselin's (1988b) empirical example relating crime to housing values and income for 49 neighborhoods in Columbus, Ohio in 1980. Monte Carlo experiments showed that these DLR tests have similar performance to their LM counterparts.

This paper focuses on similar tests but in the context of a spatial panel data model, see Lee and Yu (2010a) and Baltagi (2011) for recent surveys.[2] In fact, Baltagi, Song and Koh (2003) derived the joint LM test for spatial error correlation as well as random country effects. Additionally, they derived conditional LM tests, which test for random country effects given the presence of spatial error correlation. Also, spatial error correlation given the presence of random country effects. These conditional LM tests are an alternative to the one directional LM tests that test for random country effects ignoring the presence of spatial error correlation or the one directional LM tests for spatial error correlation ignoring the presence of random country effects. Baltagi and Liu (2008) derived a joint LM test which simultaneously tests for the absence of spatial lag dependence and random individual effects. The joint LM statistic is the sum of two standard LM statistics. The first one tests for the absence of spatial lag dependence ignoring the random individual effects, and the second one tests for the absence of random individual effects ignoring the spatial lag dependence. Baltagi and Liu (2008) also derived two conditional LM tests. The first one tests for the absence of random individual effects allowing for the possible presence of spatial lag dependence. The second one tests for the absence of spatial lag dependence allowing for the possible presence of random individual effects. Debarsey and Ertur (2010) derived LM and LR tests designed to discriminate between spatially autocorrelated disturbances versus a spatially lagged dependent variable in the context of a fixed effects spatial panel data model. Following Lee and Yu (2010b), they combine a spatial lag model with a spatially autocorrelated disturbances in a fixed effects spatial panel data setting. They derive joint, marginal as well as conditional LM and LR tests, under the assumption of normality of the disturbances. They investigate the performance of these tests using Monte Carlo experiments. This paper derives the DLR tests corresponding to the joint and conditional LM tests for spatial lag and spatial error considered by Debarsy and Ertur (2010). It illustrates these tests

[1] Few finite sample studies exist in this literature, most notably Bao and Ullah (2007) who study the finite sample properties of the maximum likelihood estimator in spatial models.

[2] Baltagi and Pirotte (2010) show that inference can be misleading if spatial dependence is ignored in spatial panel models.

using an empirical example and investigates the performance of these tests using Monte Carlo experiments.

The paper is organized as follows: Section 2 derives the DLR for the presence of spatial lag and error dependence in the context of a fixed effects panel data model. Our suggested DLR tests and their corresponding LM tests are compared using an illustrative example in section 3 and a Monte Carlo simulation in section 4. Section 5 concludes the paper.

2 The Spatial Dependence Model

Consider the spatial lag panel data regression model

$$y_t = \rho W y_t + X_t \beta + \mu + \nu_t, \qquad t = 1, ..., T, \tag{1}$$

with spatial autoregressive remainder errors

$$v_t = \lambda M \nu_t + \epsilon_t, \qquad t = 1, ..., T, \tag{2}$$

where $y_t' = (y_{t1}, \ldots, y_{tN})$ is a vector of observations on the dependent variable for N regions or households observed at time $t = 1, ..., T$. X_t is an $N \times k$ matrix of observations on k explanatory variables. We assume X_t to be of full column rank and its elements are assumed to be asymptotically bounded in absolute value. β is a $k \times 1$ vector of parameters. ρ and λ are scalar spatial autoregressive coefficients with $|\rho| < 1$ and $|\lambda| < 1$. W and M are known $N \times N$ row normalized spatial weight matrices whose diagonal elements are zero and summation of each row is 1. W and M also satisfy the condition that $(I - \lambda M)$ and $(I - \rho W)$ are non-singular. $\mu' = (\mu_1, \ldots, \mu_N)$ where μ_i denote the ith individual's fixed effect. These $\mu_i's$ are time invariant random variables that are possibly correlated with the explanatory variables. $v_t' = (v_{t1}, \ldots, v_{tN})$ is a vector of remainder disturbances that follow a first order spatial autoregressive model. $\epsilon_t' = (\epsilon_{t1}, \ldots, \epsilon_{tN})$ and ϵ_{ti} is i.i.d. over t and i and is assumed to be $N(0, \sigma_\epsilon^2)$. Normality is needed for the derivation of the DLR, see Davidson and MacKinnon (1993).

Define $E_T = I_T - \bar{J}_T$, where I_T is an identity matrix of dimension T, $\bar{J}_T = J_T/T$ and J_T is a matrix of ones of dimension T, see Baltagi (2013). Lee and Yu (2010) define $\left[F_{T,T-1}, \frac{1}{\sqrt{T}}\iota_T\right]$ as the orthonormal matrix of the eigenvectors of the demeaning operator, with $F_{T,T-1}$ the $T \times (T-1)$ matrix of eigenvectors of E_T corresponding to eigenvalues equal to 1. ι_T is a vector of ones of dimension T. Define the transformed $N \times (T-1)$ matrix $\left(y_1^*, \ldots, y_{T-1}^*\right) = (y_1, \ldots, y_T)\, F_{T,T-1}$. Equivalently, this can be written as:

$$y^* = \begin{pmatrix} y_1^* \\ \vdots \\ y_{T-1}^* \end{pmatrix} = \left(F_{T,T-1}' \otimes I_N\right) \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix},$$

where y^* is of dimension $N(T-1)\times 1$, I_N is an identity matrix of dimension N, and $\otimes$ denotes the Kronecker product. One can see that the transformed sample is shrunk by one time period. Similarly define X_t^*, v_t^* and ϵ_t^* for $t = 1, ..., T-1$. The benchmark model in Equation (1) becomes:

$$y_t^* = \rho W y_t^* + X_t^* \beta + v_t^*, \text{ for } t = 1, ..., T-1, \tag{3}$$

and

$$\nu_t^* = \lambda M \nu_t^* + \epsilon_t^*, \text{ for } \quad t = 1, ..., T-1. \tag{4}$$

Rewrite equation (3) in matrix form as

$$y^* = \rho \left(I_{T-1} \otimes W\right) y^* + X^* \beta + v^*, \tag{5}$$

and

$$\nu^* = \lambda \left(I_{T-1} \otimes M\right) \nu^* + \epsilon^*, \tag{6}$$

where X^* is $N(T-1)\times k$, β is $k\times 1$ and ε^* is $N(T-1)\times 1$. I_{T-1} is an identity matrix of dimension $T-1$. This follows from the fact that $\left(F'_{T,T-1}\otimes I_N\right)\left(I_T\otimes W\right) = \left(F'_{T,T-1}\otimes W\right) = \left(I_{T-1}\otimes W\right)\left(F'_{T,T-1}\otimes I_N\right)$. A similar result obtains when we replace W by M. The observations are ordered with t being the slow running index and i the fast running index, i.e., $y^{*\prime} = \left(y_{11}, \ldots, y_{1N}, \ldots, y_{(T-1),1}, \ldots, y_{(T-1),N}\right)$. X^*, ν^* and ϵ^* are similarly defined.

Model (5) can be rewritten as

$$\left(I_{T-1} \otimes BA\right) y^* - \left(I_{T-1} \otimes B\right) X^* \beta = \epsilon^*, \tag{7}$$

where $A = I_N - \rho W$ and $B = I_N - \lambda M$. This yields the following representation for the itth observation

$$f_{it}\left(y_{it}^*, \varphi\right) = \frac{1}{\sigma_\epsilon}\epsilon_{it}^*, \quad i = 1, \ldots, N; \quad t = 1, ..., T-1, \tag{8}$$

where

$$f_{it}\left(y_{it}^*, \varphi\right) = \frac{1}{\sigma_\epsilon}\left[y_{it}^* - \rho\sum_{s=1}^{N} w_{is} y_{st}^* - \lambda\sum_{j=1}^{N} m_{ij} y_{jt}^* + \rho\lambda\sum_{j=1}^{N}\left(m_{ij}\sum_{s=1}^{N} w_{js} y_{st}^*\right) - \left(x_{it}^* - \lambda\sum_{j=1}^{N} m_{ij} x_{jt}^*\right)'\beta\right], \tag{9}$$

with $\varphi' = \left[\beta', \sigma_\epsilon^2, \rho, \lambda\right]$.

Under the normality assumption, we have $\frac{1}{\sigma_\epsilon}\epsilon_{it}^* \sim NID(0,1)$ and hence the log-likelihood function of equation (7) is given by

$$
\begin{aligned}
L\left(y^{*},\varphi\right) &= -\frac{N\left(T-1\right)}{2}\ln 2\pi-\frac{N\left(T-1\right)}{2}\ln\sigma_{\epsilon}^{2}+\left(T-1\right)\ln\left|A\right|+\left(T-1\right)\ln\left|B\right| \\
&\quad -\frac{1}{2\sigma_{\epsilon}^{2}}\left[\left(I_{T-1}\otimes BA\right)y^{*}-\left(I_{T-1}\otimes B\right)X^{*}\beta\right]'\left[\left(I_{T-1}\otimes BA\right)y^{*}-\left(I_{T-1}\otimes B\right)X^{*}\beta\right]. \qquad (10)
\end{aligned}
$$

Ord (1975) shows that $\ln|A| = \ln|I_N - \rho W| = \sum_{i=1}^{N}\ln(1-\rho\omega_i)$, where ω_i's are the eigenvalues of W. Similarly, $\ln|B| = \ln|I_N - \lambda M| = \sum_{i=1}^{N}\ln(1-\lambda\eta_i)$, where η_i's are the eigenvalues of M. As shown in Lemma 1 of Li, Yu and Bai (2013), all the eigenvalues of a row normalized spatial weight matrix are real numbers. The Jacobian term can be rewritten as

$$
\begin{aligned}
&-\frac{N\left(T-1\right)}{2}\ln\sigma_{\epsilon}^{2}+\left(T-1\right)\ln\left|A\right|+\left(T-1\right)\ln\left|B\right| \\
&= \left(T-1\right)\sum_{i=1}^{N}\ln\left(1-\rho\omega_i\right)+\left(T-1\right)\sum_{i=1}^{N}\ln\left(1-\lambda\eta_i\right)-\frac{N\left(T-1\right)}{2}\ln\sigma_{\epsilon}^{2} \qquad (11) \\
&= \left(T-1\right)\sum_{i=1}^{N}k_{it}\left(y_{it}^{*},\varphi\right),
\end{aligned}
$$

where

$$
k_{it}\left(y_{it}^{*},\varphi\right)=\ln\left(1-\rho\omega_i\right)+\ln\left(1-\lambda\eta_i\right)-\frac{1}{2}\ln\sigma_{\epsilon}^{2},\qquad i=1,\ldots,N,\quad t=1,...,T-1. \qquad (12)
$$

For the purpose of deriving the DLR, the contribution of the ith observation to the log-likelihood function can be written as

$$
l_{it}\left(y_{it}^{*},\varphi\right)=-\frac{1}{2}\ln\left(2\pi\right)-\frac{1}{2}f_{it}^{2}\left(y_{it}^{*},\varphi\right)+k_{it}\left(y_{it}^{*},\varphi\right). \qquad (13)
$$

Defining $F_{itj}\left(y_{it}^{*},\varphi\right)=\partial f_{it}\left(y_{it}^{*},\varphi\right)/\partial\varphi_j$ and $K_{ijt}\left(y_{it}^{*},\varphi\right)=\partial k_{it}\left(y_{it}^{*},\varphi\right)/\partial\varphi_j$, then $F\left(y^{*},\varphi\right)$ and $K\left(y^{*},\varphi\right)$ are matrices with typical elements $F_{itj}\left(y_{it}^{*},\varphi\right)$ and $K_{itj}\left(y_{it}^{*},\varphi\right)$. Similarly, let $f\left(y^{*},\varphi\right)$ be the vector with typical elements $f_{it}\left(y_{it}^{*},\varphi\right)$.

For $F\left(y^{*},\varphi\right)$, the typical elements are

$$
\begin{aligned}
\frac{\partial f_{it}\left(y_{it}^{*},\varphi\right)}{\partial\beta'} &= -\frac{1}{\sigma_{\epsilon}}\left(x_{it}^{*}-\lambda\sum_{j=1}^{N}m_{ij}x_{jt}^{*}\right)', \qquad (14) \\
\frac{\partial f_{it}\left(y_{it}^{*},\varphi\right)}{\partial\sigma_{\epsilon}} &= -\frac{1}{\sigma_{\epsilon}^{2}}\left[y_{it}^{*}-\rho\sum_{s=1}^{N}w_{is}y_{st}^{*}-\lambda\sum_{j=1}^{N}m_{ij}y_{jt}^{*}+\rho\lambda\sum_{j=1}^{N}\left(m_{ij}\sum_{s=1}^{N}w_{js}y_{st}^{*}\right)-\left(x_{it}^{*}-\lambda\sum_{j=1}^{N}m_{ij}x_{jt}^{*}\right)'\beta\right], \\
\frac{\partial f_{it}\left(y_{it}^{*},\varphi\right)}{\partial\rho} &= -\frac{1}{\sigma_{\epsilon}}\left[\sum_{s=1}^{N}w_{is}y_{st}^{*}-\lambda\sum_{j=1}^{N}\left(m_{ij}\sum_{s=1}^{N}w_{js}y_{st}^{*}\right)\right], \\
\frac{\partial f_{it}\left(y_{it}^{*},\varphi\right)}{\partial\lambda} &= -\frac{1}{\sigma_{\epsilon}}\left[\sum_{j=1}^{N}m_{ij}y_{jt}^{*}-\rho\sum_{j=1}^{N}\left(m_{ij}\sum_{s=1}^{N}w_{js}y_{st}^{*}\right)-\sum_{j=1}^{N}m_{ij}x_{jt}^{*\prime}\beta\right].
\end{aligned}
$$

For $K(y^*, \varphi)$, the typical elements are

$$\begin{aligned}
\frac{\partial k_{it}(y_{it}^*, \varphi)}{\partial \beta'} &= 0, \\
\frac{\partial k_{it}(y_{it}^*, \varphi)}{\partial \sigma_\epsilon} &= -\frac{1}{\sigma_\epsilon}, \\
\frac{\partial k_{it}(y_{it}^*, \varphi)}{\partial \rho} &= -\frac{\omega_i}{1-\rho\omega_i}, \\
\frac{\partial k_{it}(y_{it}^*, \varphi)}{\partial \lambda} &= -\frac{\eta_i}{1-\lambda\eta_i}.
\end{aligned} \tag{15}$$

The DLR can be written as an artificial regression with $2N(T-1)$ observations:

$$\begin{bmatrix} f(y^*, \varphi) \\ \iota_{N(T-1)} \end{bmatrix} = \begin{bmatrix} -F(y^*, \varphi) \\ K(y^*, \varphi) \end{bmatrix} b + \text{residuals}, \tag{16}$$

where $\iota_{N(T-1)}$ denotes a vector of ones of dimension $N(T-1)$. The basic result in Davidson and MacKinnon (1984) is that the information matrix can be expressed as

$$\mathfrak{I}(y^*, \varphi) = \underset{N\to\infty}{plim} \left[\frac{1}{N} \left(F(y^*, \varphi)' F(y^*, \varphi) + K(y^*, \varphi)' K(y^*, \varphi) \right) \right]. \tag{17}$$

Another important observation is that the gradient of the log-likelihood function can be written as

$$g(y^*, \varphi) \equiv \frac{\partial \ln L(y^*, \varphi)}{\partial \varphi} = -F(y^*, \varphi)' f(y^*, \varphi) + K(y^*, \varphi)' \iota_{N(T-1)}. \tag{18}$$

The LM test statistic in score form is given by

$$g(y^*, \tilde{\varphi})' \left[N\mathfrak{I}(y^*, \tilde{\varphi}) \right]^{-1} g(y^*, \tilde{\varphi}), \tag{19}$$

where $g(y^*, \tilde{\varphi})$ is the gradient evaluated at the restricted estimates. The DLR variant of the LM test then uses a consistent estimate of the information matrix under the null of the form

$$\frac{1}{N} \left(F(y^*, \tilde{\varphi})' F(y^*, \tilde{\varphi}) + K(y^*, \tilde{\varphi})' K(y^*, \tilde{\varphi}) \right). \tag{20}$$

In this case, the LM test coincides with the explained sum of squares from the DLR regression in Equation (16). Equation (16) evaluated at the restricted estimates can be written as

$$\mathcal{Y} = \mathcal{X} b + \text{residuals}, \tag{21}$$

where $\mathcal{Y} = \begin{bmatrix} f(y^*, \tilde{\varphi}) \\ \iota_{N(T-1)} \end{bmatrix}$ and $\mathcal{X} = \begin{bmatrix} -F(y^*, \tilde{\varphi}) \\ K(y^*, \tilde{\varphi}) \end{bmatrix}$. Since the OLS estimator of b is $(\mathcal{X}'\mathcal{X})^{-1}\mathcal{X}'\mathcal{Y}$, the explained sum of squares from the DLR regression is $\mathcal{Y}'\mathcal{X}(\mathcal{X}'\mathcal{X})^{-1}\mathcal{X}'\mathcal{Y}$. Note that the residual sum of squares of

the above artificial regression is $\mathcal{Y}'\left[I-\mathcal{X}\left(\mathcal{X}'\mathcal{X}\right)^{-1}\mathcal{X}'\right]\mathcal{Y}$ and $\mathcal{Y}'\mathcal{Y}=f\left(y^*,\tilde{\varphi}\right)'f\left(y^*,\tilde{\varphi}\right)+\iota_{N(T-1)'}\iota_{N(T-1)}=2N(T-1)$. Therefore, the DLR test statistics can be alternatively computed as $2N(T-1)$ minus the residual sum of squares of the above artificial regressions. This DLR test is computationally simple and requires only the eigenvalues of W. These eigenvalues are also needed for ML estimation and the LM test.

2.1 Joint DLR test for $H_0^a:\lambda=\rho=0$

Under $H_0^a:\rho=\lambda=0$, the restricted MLE are the OLS estimates of the following transformed panel data regression:

$$y^*=X^*\beta+\epsilon^*,\qquad t=1,...,T-1 \tag{22}$$

In this case, $\tilde{\beta}=\left(X^{*\prime}X^*\right)^{-1}X^{*\prime}y^*$, $\tilde{\sigma}_\epsilon^2=\frac{1}{N(T-1)}\tilde{e}^{*\prime}\tilde{e}^*$ with $\tilde{e}^*$ denoting the OLS residuals of this transformed panel data regression. Using these restricted ML estimates $\tilde{\varphi}'=\left[\tilde{\beta}',\tilde{\sigma}_\epsilon^2,\tilde{\rho},\tilde{\lambda}\right]$ with $\tilde{\rho}=\tilde{\lambda}=0$, we run the following $2N\left(T-1\right)$ observations artificial regression:

$$\begin{bmatrix}\frac{1}{\tilde{\sigma}_\epsilon}\tilde{e}^*\\ \iota_{N(T-1)}\end{bmatrix}=\begin{bmatrix}\frac{1}{\tilde{\sigma}_\epsilon}X^* & \frac{1}{\tilde{\sigma}_\epsilon^2}\tilde{e}^* & \frac{1}{\tilde{\sigma}_\epsilon}\left(I_{T-1}\otimes W\right)y^* & \frac{1}{\tilde{\sigma}_\epsilon}\left(I_{T-1}\otimes M\right)\tilde{e}^*\\ 0 & -\frac{1}{\tilde{\sigma}_\epsilon}\iota_{N(T-1)} & -\iota_{T-1}\otimes\omega & -\iota_{T-1}\otimes\eta\end{bmatrix}b+\text{residuals}, \tag{23}$$

where $\omega=\left(\omega_1,\ldots,\omega_N\right)'$ are the eigenvalues of W, and $\eta=\left(\eta_1,\ldots,\eta_N\right)'$ are the eigenvalues of M. $\iota_{N(T-1)}$ and ι_{T-1} are vector of ones of dimension $N\left(T-1\right)$ and $T-1$, respectively. The explained sum of squares from the DLR in (23) will provide an asymptotically valid test statistic for H_0^a. This should be asymptotically distributed as χ_2^2 under the null.[3] It can alternatively be computed as $2N\left(T-1\right)$ minus the residual sum of squares of the above artificial regression.[4]

[3]In spatial models, the functional form changes with the sample size, i.e., the functions F and K should have a sample size subscript. As a result, Slutsky's Lemma no longer applies and replacing the true parameter values φ with consistent estimates in (1) above does not necessarily lead to a consistent estimate of the information matrix. When F and K have the same functional form in different sample sizes, their continuity would guarantee this. Here, we need some stronger assumptions on the functions or require that the estimates are converging in a stronger sense (e.g. almost surely). Also, the standard LM tests are derived under the normality and homoskedasticity assumptions of the regression disturbances. Hence, they may not be robust against non-normality or heteroskedasticity of the disturbances. Baltagi and Yang (2013) applied the technique in Born and Breitung (2011) and introduced general methods to modify the standard LM tests so that they become robust against heteroskedasticity and non-normality. This is beyond the scope of this paper, though. We acknowledge this limitation in the paper and thank the referee for pointing it out.

[4]It is important to point out that the asymptotic distribution of our test statistics are not explicitly derived in the paper. There is no proof in the literature that the LM tests are asymptotically χ^2 under the null. Given the simulations below, they most likely are but we cannot say under what assumptions this holds. These are likely to hold under a similar set of primitive assumptions developed by Kelejian and Prucha (2001) for the Moran-I test.

2.2 Conditional DLR Test for $H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ)

Under $H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ), the restricted model is a panel data transformed regression with first order spatial autoregressive disturbances:

$$\begin{aligned} y^* &= X^*\beta + v^*, \\ v^* &= \lambda\left(I_{T-1} \otimes M\right) v^* + \epsilon^*. \end{aligned} \tag{24}$$

Let $\hat{\beta}$, $\hat{\lambda}$ and $\hat{\sigma}_\epsilon^2$ denote the MLEs of β, λ and σ_ϵ^2 under this restricted model. Using these restricted ML estimates $\hat{\varphi}' = \left[\hat{\beta}', \hat{\sigma}_\epsilon^2, \hat{\rho}, \hat{\lambda}\right]$ with $\hat{\rho} = 0$, we run the following $2N\left(T-1\right)$ observations artificial regression:

$$\begin{bmatrix} \frac{1}{\hat{\sigma}_\epsilon}\hat{e}^* \\ \iota_{N(T-1)} \end{bmatrix} = \begin{bmatrix} \frac{1}{\hat{\sigma}_\epsilon}\left(I_{T-1} \otimes \hat{B}\right) X^* & \frac{1}{\hat{\sigma}_\epsilon^2}\hat{e}^* & \frac{1}{\hat{\sigma}_\epsilon}\left(I_{T-1} \otimes W\hat{B}\right) y^* & \frac{1}{\hat{\sigma}_\epsilon}\left(I_{T-1} \otimes M\right)\hat{u}^* \\ 0 & -\frac{1}{\hat{\sigma}_\epsilon}\iota_{N(T-1)} & -\iota_{T-1} \otimes \omega & -\iota_{T-1} \otimes \eta^* \end{bmatrix} b + \text{residuals}, \tag{25}$$

where $\hat{e}^* = \left(I_{T-1} \otimes \hat{B}\right)\hat{v}^*$ with $\hat{B} = I_N - \hat{\lambda}M$ and $\hat{v}^* = y^* - X^*\hat{\beta}$, $\omega = \left(\omega_1, \ldots, \omega_N\right)'$ and $\eta^* = \left(\frac{\eta_1}{1-\hat{\lambda}\eta_1}, \ldots, \frac{\eta_N}{1-\hat{\lambda}\eta_N}\right)'$. The explained sum of squares from the DLR in (25) will provide an asymptotically valid test statistic for H_0^b. This should be asymptotically distributed as χ_1^2 under the null. It can alternatively be computed as $2N\left(T-1\right)$ minus the residual sum of squares of the above artificial regression.

2.3 Conditional DLR Test for $H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ)

Under $H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ), the restricted model is a panel spatial autoregressive regression model of order one:

$$y^* = \rho\left(I_{T-1} \otimes W\right) y^* + X^*\beta + \epsilon^*. \tag{26}$$

Let $\bar{\beta}$, $\bar{\rho}$ and $\bar{\sigma}_\epsilon^2$ denote the MLEs of β, ρ and σ_ϵ^2 under this restricted model. Using these restricted ML estimates $\bar{\varphi}' = \left[\bar{\beta}', \bar{\sigma}_\epsilon^2, \bar{\rho}, \bar{\lambda}\right]$ with $\bar{\lambda} = 0$, we run the following $2N\left(T-1\right)$ observations artificial regression:

$$\begin{bmatrix} \frac{1}{\bar{\sigma}_\epsilon}\bar{e}^* \\ \iota_{N(T-1)} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{\sigma}_\epsilon}X^* & \frac{1}{\bar{\sigma}_\epsilon^2}\bar{e}^* & \frac{1}{\bar{\sigma}_\epsilon}\left(I_{T-1} \otimes W\right) y^* & \frac{1}{\bar{\sigma}_\epsilon}\left(I_{T-1} \otimes M\right)\bar{e}^* \\ 0 & -\frac{1}{\bar{\sigma}_\epsilon}\iota_{N(T-1)} & -\iota_{T-1} \otimes \omega^* & -\iota_{T-1} \otimes \eta \end{bmatrix} b + \text{residuals}, \tag{27}$$

where $\bar{e}^* = \left(I_{T-1} \otimes \bar{A}\right) y^* - X^*\hat{\beta}$ with $\bar{A} = I_N - \bar{\rho}W$, $\omega^* = \left(\frac{\omega_1}{1-\bar{\rho}\omega_1}, \ldots, \frac{\omega_N}{1-\bar{\rho}\omega_N}\right)'$ and $\eta = \left(\eta_1, \ldots, \eta_N\right)'$. The explained sum of squares from the DLR in (27) will provide an asymptotically valid test statistic for

H_0^c. This should be asymptotically distributed as χ_1^2 under the null. It can alternatively be computed as $2N(T-1)$ minus the residual sum of squares of the above artificial regression.

3 Empirical Illustration

Following Munnell (1990), Baltagi and Pinnoi (1995) considered the Cobb-Douglas production function relationship investigating the contribution of different types of public infrastructure on private production. Their regression model is as follows:

$$\log(Y) = \alpha + \beta_1 \log(K_1) + \beta_2 \log(K_2) + \beta_3 \log(L) + \beta_4 \log(Unemp) + u,$$

where Y is gross state product, K_1 is public capital which includes highways and streets, water and sewer facilities and other public buildings and structures. K_2 is the private capital stock based on the Bureau of Economic Analysis national stock estimates, L is labor input measured as employment in nonagricultural payrolls. $Unemp$ is the state unemployment rate included to capture business cycle effects. This panel consists of annual observations for 48 contiguous states over the period 1970-1986. The weighting matrix $W = M$ has elements different from zero if two states are neighbors. According to the queen contiguity matrix, Arizona and Colorado are considered neighbors. This weighting matrix has been row-normalized. Table 1 compares the results from applying the DLR statistics derived in this paper with their LM counterparts derived by Debarsy and Ertur (2010). The DLR statistics are computed as $2N(T-1)$ minus the residual sum of squares from (23), (25) and (27). As we can see from the table, for all hypotheses considered, the DLR and its LM counterpart are close and provide the same decision. The joint test and conditional tests reject the absence of spatial dependence.

4 Monte Carlo Simulation

This section investigates the small sample performance of the DLR and LM tests. Following Debarsy and Ertur (2010), we generate the data using the model in Equation (1) with one regressor x_{it} generated by $x_{it} = 0.1t + 0.5x_{i,t-1} + z_{it}$ with $z_{it} \overset{iid}{\sim} U[-5,5]$. The true parameters are as follows: $\beta = 3$, while ρ and λ varied over the range $(-0.2, -0.5, -0.8, 0, 0.2, 0.5, 0.8)$. The sample size chosen is $(N = 49; T = 10)$. The weighting matrix $W = M$ is a row-normalized Rook-type of order 2. The individual specific effects are $\mu_i \overset{iid}{\sim} U[-5,5]$ and the disturbances are $\epsilon_{it} \overset{iid}{\sim} N(0,1)$. We performed 1,000 replications. It is worth pointing out that the eigenvalues of W need only to be computed once. Table 2 shows the simulation results. The size of the joint DLR and LM tests for $H_0^a : \rho = \lambda = 0$, using the 5% critical value of a χ_2^2, is 5.8%

and 5.5%, respectively. The simulations are performed using a 2.83GHz processor desktop computer with 2GB of RAM and running Windows 7 Enterprise, Service Pack 1. Computation time of the simulation results is shown in Table 3. For the joint test, DLR computation time in seconds is more much shorter than LM. For the conditional tests, the computation time for the LM and DLR are similar. Figure 1 plots the corresponding power of the joint DLR and LM tests for $H_0^a : \rho = \lambda = 0$. The size of the conditional LM test for $H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ), using the 5% critical value of a χ_1^2, varies between 4.8% and 7.0% depending on the value of λ, while that of the corresponding DLR test varies between 4.7% and 7.2% depending on the value of λ. Figure 2 plots the corresponding power of the conditional LM test for $H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ). Similarly, the size of the conditional LM test for $H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ), using the 5% critical value of a χ_1^2, varies between 3.8% and 6.3% depending on the value of ρ, while that of the corresponding DLR test varies between 3.9% and 6.4% depending on the value of ρ. Figure 3 plots the corresponding power of the conditional LM test for for $H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ). As clear from the figures, the joint and conditional DLR and LM tests yield similar small sample performance in the Monte Carlo experiments.

5 Conclusion

This paper derives three artificial DLR tests corresponding to the LM tests derived by Debarsy and Ertur (2010) for the fixed effects spatial lag regression model with spatial auto-regressive error. The first DLR jointly tests for zero spatial lag dependence as well as zero spatial autoregressive error in a fixed effects panel data model. The second DLR conditionally tests for zero spatial lag dependence allowing for spatial error dependence. While the third DLR conditionally tests for zero spatial error dependence allowing for spatial lag dependence. The proposed tests are illustrated using the productivity puzzle empirical example by Munnell (1990). In addition, Monte Carlo experiments show that the small sample performance of these DLR tests have similar performance to their corresponding LM counterparts. Furthermore, it would be nice if the normality assumption of the disturbances can be relaxed, though this is beyond the scope of this paper.

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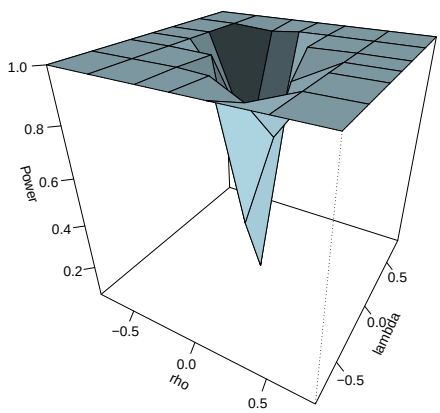
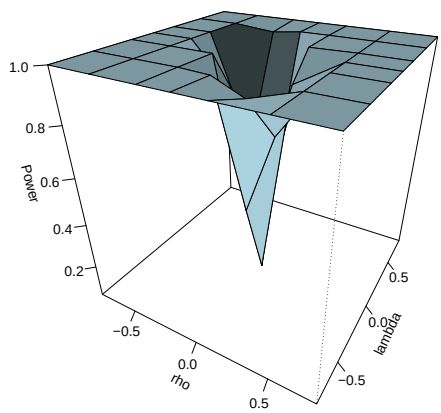
Table 1: Empirical Test Results

	Statistic	p-value
$H_0^a : \lambda = \rho = 0$		
$LM_{\lambda\rho}$	243.405	0.000
$DLR_{\lambda\rho}$	191.157	0.000
$H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ)		
$LM_{\rho\|\lambda}$	5.960	0.015
$DLR_{\rho\|\lambda}$	6.133	0.013
$H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ)		
$LM_{\lambda\|\rho}$	34.326	0.000
$DLR_{\lambda\|\rho}$	34.495	0.000

Figure 1: Test for $H_0^a : \lambda = \rho = 0$



(a) LM Test



(b) DLR Test

Table 2: Simulation Results

ρ	λ	$LM_{\lambda\rho}$	$DLR_{\lambda\rho}$	$LM_{\rho\|\lambda}$	$DLR_{\rho\|\lambda}$	$LM_{\lambda\|\rho}$	$DLR_{\lambda\|\rho}$
-0.8	-0.8	1.000	1.000	1.000	1.000	1.000	1.000
	-0.5	1.000	1.000	1.000	1.000	0.953	0.961
	-0.2	1.000	1.000	1.000	1.000	0.338	0.369
	0	1.000	1.000	1.000	1.000	0.038	0.039
	0.2	1.000	1.000	1.000	1.000	0.406	0.378
	0.5	1.000	1.000	1.000	1.000	0.994	0.994
	0.8	1.000	1.000	1.000	1.000	1.000	1.000
-0.5	-0.8	1.000	1.000	1.000	1.000	1.000	1.000
	-0.5	1.000	1.000	1.000	1.000	0.953	0.960
	-0.2	1.000	1.000	1.000	1.000	0.319	0.354
	0	1.000	1.000	1.000	1.000	0.045	0.046
	0.2	1.000	1.000	1.000	1.000	0.432	0.405
	0.5	0.999	0.999	1.000	1.000	0.994	0.994
	0.8	1.000	1.000	0.999	0.999	1.000	1.000
-0.2	-0.8	1.000	1.000	0.881	0.888	1.000	1.000
	-0.5	1.000	1.000	0.839	0.852	0.964	0.971
	-0.2	0.994	0.999	0.803	0.808	0.288	0.327
	0	0.797	0.825	0.724	0.726	0.056	0.060
	0.2	0.586	0.584	0.689	0.692	0.427	0.407
	0.5	0.988	0.985	0.574	0.575	0.989	0.988
	0.8	1.000	1.000	0.520	0.522	1.000	1.000
0	-0.8	1.000	1.000	0.060	0.060	1.000	1.000
	-0.5	0.993	0.995	0.049	0.049	0.971	0.975
	-0.2	0.362	0.418	0.049	0.049	0.344	0.382
	0	0.058	0.055	0.048	0.049	0.063	0.064
	0.2	0.569	0.523	0.049	0.049	0.435	0.408
	0.5	0.999	0.999	0.070	0.072	0.992	0.989
	0.8	1.000	1.000	0.049	0.047	1.000	1.000
0.2	-0.8	1.000	1.000	0.922	0.920	1.000	1.000
	-0.5	0.934	0.946	0.869	0.869	0.969	0.978
	-0.2	0.754	0.754	0.808	0.813	0.362	0.388
	0	0.909	0.898	0.741	0.747	0.047	0.048
	0.2	0.996	0.996	0.672	0.678	0.406	0.376
	0.5	1.000	1.000	0.568	0.567	0.991	0.990
	0.8	1.000	1.000	0.575	0.570	1.000	1.000
0.5	-0.8	1.000	1.000	1.000	1.000	1.000	1.000
	-0.5	1.000	1.000	1.000	1.000	0.976	0.979
	-0.2	1.000	1.000	1.000	1.000	0.351	0.377
	0	1.000	1.000	1.000	1.000	0.046	0.041
	0.2	1.000	1.000	1.000	1.000	0.417	0.396
	0.5	1.000	1.000	0.997	0.998	0.989	0.988
	0.8	1.000	1.000	1.000	1.000	1.000	1.000
0.8	-0.8	1.000	1.000	0.999	0.997	1.000	1.000
	-0.5	1.000	1.000	1.000	1.000	0.989	0.990
	-0.2	1.000	1.000	1.000	1.000	0.379	0.396
	0	1.000	1.000	1.000	1.000	0.040	0.042
	0.2	1.000	1.000	1.000	1.000	0.454	0.434
	0.5	1.000	1.000	1.000	1.000	0.993	0.992
	0.8	1.000	1.000	1.000	1.000	1.000	1.000

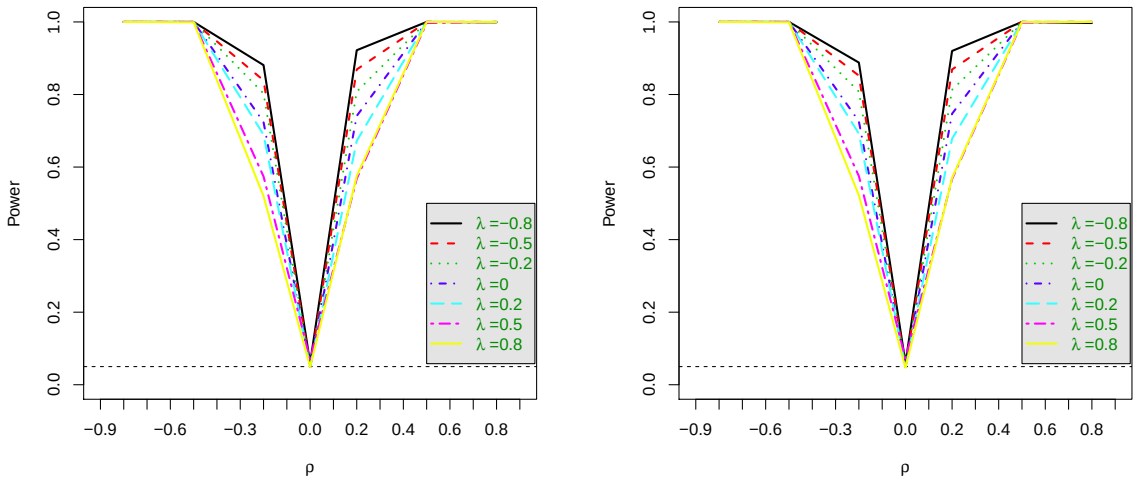
Notes: 1,000 replications

Table 3: Compuation Time of Simulation Results

ρ	λ	$LM_{\lambda\rho}$	$DLR_{\lambda\rho}$	$LM_{\rho\|\lambda}$	$DLR_{\rho\|\lambda}$	$LM_{\lambda\|\rho}$	$DLR_{\lambda\|\rho}$
-0.8	-0.8	154.60	41.94	542.95	539.57	673.19	675.59
	-0.5	148.48	30.53	544.75	542.49	674.80	675.94
	-0.2	148.80	30.47	544.40	542.10	674.16	678.12
	0	148.25	30.30	544.70	541.72	673.82	675.95
	0.2	148.26	30.59	545.35	543.84	674.69	676.14
	0.5	147.86	30.64	544.80	542.94	676.17	676.93
	0.8	148.26	30.64	544.74	542.79	674.67	677.79
-0.5	-0.8	148.47	30.68	546.33	542.01	675.76	679.71
	-0.5	148.74	30.84	547.64	544.32	673.79	676.17
	-0.2	148.17	30.51	544.94	541.81	673.86	677.78
	0	148.08	30.44	544.64	542.35	674.36	676.28
	0.2	148.40	30.57	546.45	541.84	673.88	676.43
	0.5	196.23	30.67	545.19	541.97	684.14	687.69
	0.8	150.58	31.00	552.47	550.78	679.82	687.33
-0.2	-0.8	148.87	31.25	548.63	544.34	677.87	693.14
	-0.5	149.06	30.53	546.89	545.52	675.74	678.69
	-0.2	148.59	30.77	546.10	543.35	677.67	678.02
	0	148.48	30.53	546.45	543.23	675.36	678.24
	0.2	148.53	31.41	546.88	561.44	674.94	677.40
	0.5	149.51	30.52	545.38	542.27	674.32	676.56
	0.8	148.40	30.65	545.27	566.47	744.56	677.72
0	-0.8	148.16	30.32	545.14	542.01	674.10	676.78
	-0.5	148.28	30.53	546.05	541.98	674.27	676.84
	-0.2	148.44	30.34	545.13	542.47	675.45	676.98
	0	148.17	30.31	545.08	541.71	673.98	677.48
	0.2	148.16	30.29	544.86	541.79	674.01	676.61
	0.5	148.19	30.52	546.13	542.79	674.62	676.86
	0.8	148.41	30.33	545.12	542.29	674.92	676.40
0.2	-0.8	148.61	30.69	545.20	542.12	674.49	677.77
	-0.5	157.19	43.37	555.58	547.00	682.05	684.23
	-0.2	150.21	31.62	552.50	560.22	698.88	731.89
	0	275.49	30.71	545.99	541.87	682.82	687.41
	0.2	150.31	32.19	563.30	553.04	685.14	680.04
	0.5	149.47	31.19	554.56	571.71	738.19	715.10
	0.8	148.28	30.83	542.95	543.64	675.61	677.32
0.5	-0.8	147.90	31.00	543.07	543.73	676.69	679.20
	-0.5	148.28	30.84	554.07	555.43	686.90	678.99
	-0.2	147.67	30.96	542.35	541.09	673.85	675.77
	0	147.46	30.66	541.37	541.43	675.33	676.45
	0.2	148.01	30.93	542.52	542.56	675.55	677.72
	0.5	148.01	30.85	542.45	542.84	675.54	676.97
	0.8	147.95	30.98	542.45	543.11	675.06	676.42
0.8	-0.8	147.82	30.89	542.15	542.72	679.29	693.55
	-0.5	151.58	31.00	542.62	542.97	675.73	677.56
	-0.2	148.22	30.90	547.90	550.36	685.30	678.20
	0	147.76	31.06	544.35	544.20	677.93	676.78
	0.2	148.03	30.94	543.32	543.01	675.34	677.74
	0.5	148.01	31.01	542.88	542.76	675.22	677.35
	0.8	148.16	30.87	544.19	544.46	676.66	678.98

Notes: The simulation is performed using a 2.83GHz processor desktop computer with 2GB of RAM and running Windows 7 Enterprise, Service Pack 1. 1,000 replications. Unit in seconds.

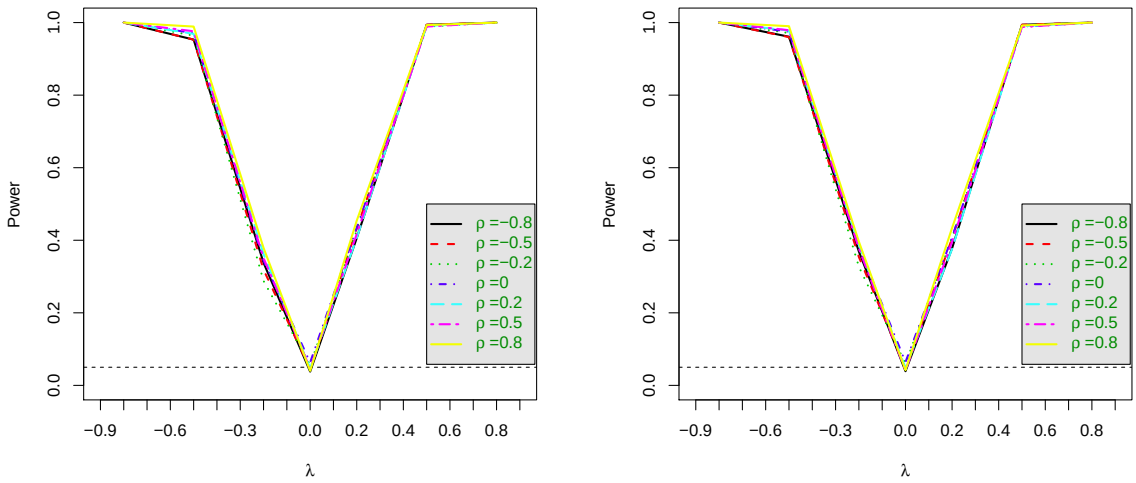
Figure 2: Test for $H_0^b : \rho = 0$ (allowing an unrestricted estimate of λ)



(a) LM Test

(b) DLR Test

Figure 3: Test for $H_0^c : \lambda = 0$ (allowing an unrestricted estimate of ρ)



(a) LM Test

(b) DLR Test